University of California at Berkeley
Department of Physics
Physics 8A, Spring 2008

Final Exam
May 20, 2008 5:00 PM

You will be given 170 minutes to work this exam. No books, but you may use a double-sided, handwritten note sheet no larger than an 8 1/2 by 11 sheet of paper. No electronics of any kind (calculator, cell phone, iPod, etc) are allowed.

Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Write your answers directly on the exam, and if you have to use the back of a sheet make sure to put a note on the front. Do not use a blue book or scratch paper.

Each part is worth the number of points indicated. These should sum to 200 points. Setup and explanation are worth almost all of the points. Clearly state what you are doing and why. In particular, make sure that you explain what principles and conservation rules you are applying, and how they relate.

There are two pages of info *at the back.* You can tear them off and keep them separate if you'd like.

NAME: Mike DeWeese

SID NUMBER: ____________

DISCUSSION SECTION NUMBER: ____________

DISCUSSION SECTION DATE/TIME: ____________

1	
2	
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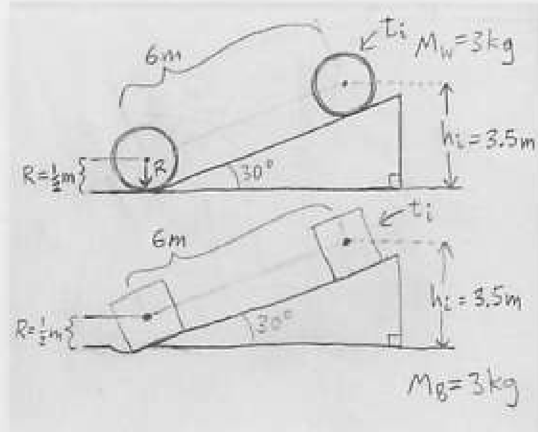
6	
Total	

Read the problems carefully.
Try to do all the problems.
If you get stuck, go on to the next problem.
Don't give up! Try to remain relaxed and work steadily.

Problem 1 (45points) Rolling vs. Sliding.



A 3 kg wheel of radius $R = 0.5$ m rolls without slipping down a ramp that makes an angle of 30° with the ground. The wheel starts from rest with its center of mass at a height $h_i = 3.5$ m above the ground. The ramp has a coefficient of kinetic friction $\mu_k = 1/3$. You may use $g = 10$ m/s^2, sin(30 °) = ½, and cos(30°) = 0.9.

a) How much gravitational potential energy does the wheel lose by the time it reaches the ground?
b) What is the wheel's *total* kinetic energy (including rotational KE) when it reaches the ground?
c) What is the relationship between the wheel's angular velocity, ω, and its linear velocity, v?
d) If the wheel's moment of inertia is $I = MR^2$, then at any given time what fraction of the wheel's total kinetic energy is its linear kinetic energy?
e) Now draw a free body diagram for a 3 kg *block* sliding down an identical ramp.
f) What is the magnitude of the normal force on the block?
g) What is the magnitude of the force of friction acting on the sliding block?
h) How much work is done by friction on the block as it slides 6m along the ramp? Check signs.
i) What is the block's kinetic energy at the bottom of the ramp? Is the wheel or block faster at the bottom?

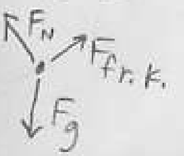
a) $\Delta PE_g = mgh_f - mgh_i = 3kg \cdot 10\frac{m}{s^2}(0.5m - 3.5m) = -90j \Rightarrow \boxed{\text{wheel loses } 90j \text{ of } PE_g}$

b) by cons. of E: $(KE_{total} + PE_g)_f = (KE_{tot.} + PE_g)_i \Rightarrow \Delta KE_{tot} = -\Delta PE_g \Rightarrow \boxed{KE_{tot\,f} = +90j}$

c) x "+" $\boxed{v = R\omega}$

d) $KE_L = \frac{1}{2}mv^2$ $KE_R = \frac{1}{2}I\omega^2 = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}mv^2 = KE_L \Rightarrow KE_{tot} = 2KE_L \Rightarrow KE_L = \boxed{\frac{1}{2}KE_{tot}}$

e)

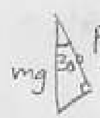


f)

N2L: y: $F_{net\,y} = m\cancel{a_y}^{0}$

$F_N - 0.9mg = 0$

$F_N = 0.9mg = .9\ 3kg\ 10\frac{m}{s^2} = \boxed{27N}$



$F_{g_y} = -\cos(30°)mg = -.9mg$

g) kinetic Friction: $F_{fr_k} = \mu_k F_N = \frac{1}{3}27N = \boxed{9N}$

h) $W_{\text{on block}} = -\vec{F}_{on} \cdot \Delta\vec{x} = -9N \cdot 6m = \boxed{-54\,Nm} = -54j$

i) $(KE_f + PE_f) - (KE_i + PE_i) = W_{on}$

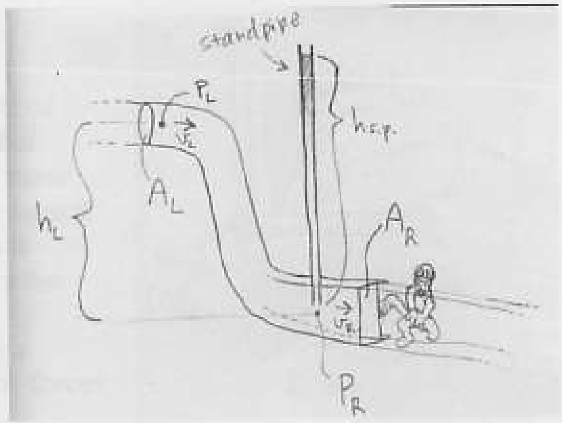
$KE_f = -PE_f + PE_i - \cancel{KE_i}^{0} - 54j = 90j - 54j = \boxed{36j}$ ← for the block

The wheel has $\frac{1}{2} \cdot 90j = 45j$ at bottom so $\boxed{\text{wheel}}$ is faster

(since $KE_{linear} = \frac{1}{2}mv^2 \Rightarrow |v|$ goes up with increasing KE.)

Problem 2 (30 points) Water in a pipe.

Water at a gauge pressure of $P_L = 5 \times 10^4$ N/m^2 is flowing with speed $v_L = 10$ m/s into the left end of a pipe, which is elevated above the right side of the pipe by $h = 15$ m. The left end of the pipe has a cross-sectional area $A_L = 1$ m^2; but the width of the right side of the pipe can be adjusted to get a range of values for A_R. A "standpipe" is inserted into the right side of the pipe. No water is flowing into or out of the standpipe, which is open to the atmosphere on the top. The density of water is 1000 kg/m^3. You may ignore viscous drag forces.



a) What volume of water passes through the pipe in 10 seconds?
b) If $A_R = A_L$, then how fast is water flowing on the right side of the pipe?
c) In that case, what is the pressure, P_R, on the right side? Please state whether you are using gauge or absolute pressure.
d) In that case, how high will the water level be in the stand pipe?
e) Now a workman adjusts the width of the pipe on the right side so that $P_L = P_R$. Find the water speed, v_R, that would be required on the right side in order to make $P_L = P_R$.
f) In order to achieve this, what value for ~~A_L~~ A_R is required to make $P_L = P_R$?

a) $V_{water} = v_L \cdot A_L \cdot \Delta t = 10\frac{m}{s} \cdot 1m^2 \cdot 10s = \boxed{100 m^3}$

b) continuity eq. $A_L v_L = A_R v_R \Rightarrow v_R = v_L \cdot \frac{A_L}{A_R} = v_L \frac{A_L}{A_L} = v_L = \boxed{10\frac{m}{s}}$

c) Bernoulli's eq.: $P_L + \rho g h_L + \frac{1}{2}\rho v_L^2 = P_R + \cancel{\rho g h_R}^{0} + \underbrace{\frac{1}{2}\rho v_R^2}_{=\frac{1}{2}\rho v_L^2}$

I'll use gauge pressure: $P_R = P_L + \rho g h_L = 5\times10^4 \frac{N}{m^2} + 10^3\frac{kg}{m^3} \cdot 10\frac{m}{s^2} \cdot 15m$

$= (5+15)\times10^4 \frac{N}{m^2}$

$= 20\times10^4 \frac{N}{m^2} = \boxed{2\times10^5 \frac{N}{m^2}}$

d) $P_{\text{Bottom s.p.}} = P_R \qquad P_{\text{top s.p.}} = 0\frac{N}{m^2}$ (gauge pressure) $\quad v_{\text{bottom s.p.}} = v_{\text{top s.p.}} = 0$

Bernouli's eq: $\cancel{P_{\text{top s.p.}}}^{0} + \rho g h_{s.p.} = P_{\text{bottom s.p.}} + \cancel{\rho g h_{\text{bottom}}}^{0}$

$h_{s.p.} = \frac{P_R}{\rho g} = \frac{2\times10^5 \frac{N}{m^2}}{10^3\frac{kg}{m^3} \, 10 \frac{m}{s^2}} = \boxed{20 m}$

e) $P_L = P_R$: Bernouli's eq.: $\cancel{P_L} + \rho g h_L + \frac{1}{2}\rho v_L^2 = \cancel{P_R}^{P_L} + \cancel{\rho g h_R}^{0} + \frac{1}{2}\rho v_R^2$

$\cancel{\rho} g h_L + \frac{1}{2}\cancel{\rho} v_L^2 = \frac{1}{2}\cancel{\rho} v_R^2$

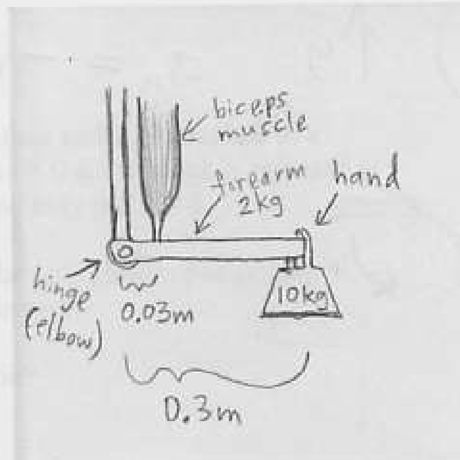
$v_R = \sqrt{2gh_L + v_L^2} = \sqrt{2\cdot10\frac{m}{s^2}\cdot15m + (10\frac{m}{s})^2} = \sqrt{300+100}\,\frac{m}{s}$

$= \sqrt{400}\,\frac{m}{s} = \boxed{20\frac{m}{s}}$

f) $A_L v_L = A_R v_R \Rightarrow A_R = A_L \frac{v_L}{v_R} = 1m^2 \cdot \frac{10 m/s}{20 m/s} = \boxed{\frac{1}{2} m^2}$

Problem 3 (35 points) Arm curls and torque.

A simple model of an arm is depicted at the right. It consists of vertical bar connected via a hinge at the elbow to a second bar representing the forearm, which is $L_{fa} = 0.3$ m long and has mass $M_{fa} = 2$ kg equally distributed across its length. The biceps muscle connects to the forearm 0.03 m from the hinge, and the arm is supporting a $M_w = 10$ kg weight. The moment of inertia of a bar about one end is $I = ML^2/3$.



a) Make a free body diagram of the forearm, and make a second, extended diagram showing where each of the forces is acting on the forearm.

b) Using the hinge as your pivot point, what is the torque due to the 10 kg weight? Please indicate your convention for positive torque.

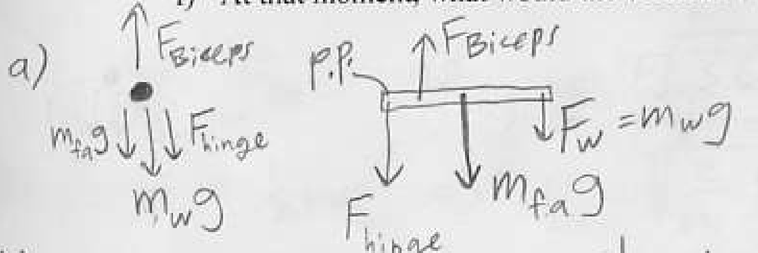
c) What force must be exerted by the biceps muscle on the forearm to keep the arm from moving?

d) What is the magnitude and direction of the force on the forearm from the vertical bar?

e) If the 10 kg weight is let go, what would the angular acceleration of the forearm be immediately after the weight is dropped assuming the biceps continue to exert the same force as before?

f) At that moment, what would the vertical acceleration of the hand be?

a)



"+" (I'll define clockwise rotations as positive)

$\uparrow y$ (I'll define the positive y direction as upward)

b) $\tau_w = \vec{r}_{h\to w} \times \vec{F}_w = r_{h\to w} F_w \sin(90°) = +0.3\text{m} \cdot 10\text{kg}\, 10\frac{\text{m}}{\text{s}^2} = \boxed{30 \text{ Nm}}$

c) N2L$_\theta$: $\tau_{net} = I\cancel{\alpha}^{\,0}$ static!

$\tau_w + \tau_b + \cancel{\tau_h}^{\,0} + \tau_{fa} = 0$

$\tau_{fa} = +\frac{L_{fa}}{2} \cdot M_{fa}$

$+30\text{ Nm} - F_B \cdot 0.03\text{m} + 0 + 2\text{kg} \cdot 10\frac{\text{m}}{\text{s}^2} \cdot .15\text{m} = 0$

$F_B = \frac{30\text{N} + 3\text{N}}{0.03} = \frac{33\text{N}}{0.03} = \frac{3300\text{N}}{3} = \boxed{1100\text{N}}$

d) N2L: $\uparrow y$ y: $F_{hinge} + F_w + F_{f.a.} + F_B = m\cancel{a_y}^{\,0}$

(F_{hinge} this is Force on Forearm from vert. bar)

$F_{hinge} - 10\text{kg}\,10\frac{\text{m}}{\text{s}^2} - 10\frac{\text{m}}{\text{s}^2}\,2\text{kg} + 1100\text{N} = 0$

$F_{hinge} = +100\text{N} + 20\text{N} - 1100\text{N} = \boxed{-980\text{ N}}$ downward

$\approx -1000\text{N}$

e) N2L$_\theta$: $\tau_{net} = I\alpha$

$\tau_{fa} + \tau_B + \cancel{\tau_h}^{\,0} = \frac{m_{fa} L_{fa}^2}{3} \alpha$

$+10\frac{\text{m}}{\text{s}^2} 0.15\text{m} \cdot 2\text{kg} - 1100\text{N} \cdot 0.03\text{m} = \frac{2\text{kg}(.3)^2\text{m}^2}{3} \alpha$

$+3\text{ Nm} - 33\text{ Nm} = 2(0.03)\text{m}^2\text{kg}\,\alpha$

$\alpha = \frac{-30\text{ Nm}}{2(0.03)\text{m}^2\text{kg}} = \frac{-10}{2(0.01)}\text{s}^{-2} = \frac{-10}{0.02}\text{s}^{-2} = \boxed{-500\text{ s}^{-2}}$

(please see back)

f) $\uparrow y$ $\quad a_y = -L_{f.a.}\,\alpha = -0.3\,m \cdot (-500\,s^{-2})$

$= \boxed{+150\,\frac{m}{s^2}}$ (That's 15 "g's"!)

Problem 4 (25 points) Harmonic oscillator.

A block with a mass of $M = 2$ kg rests on a frictionless horizontal surface and it is attached to a (massless & frictionless) spring with spring constant $k = 18$ N/m. At $t = 0$ s, the block is released at rest from position $\Delta x = 2$ m from its equilibrium position, $\Delta x = 0$. You may use $\pi \approx 3$.

a) How much potential energy is stored in the spring at $t = 0$?
b) What is the kinetic energy of the mass right when it reaches the equilibrium position?
c) What is the speed of the mass as it reaches the equilibrium position?
d) What is the angular frequency, ω, of the block's oscillation?
e) At what time does the block first cross the equilibrium position?

a) $PE_{sp} = \frac{1}{2} k \Delta x^2 = \frac{1}{2}\, 18 \frac{N}{m} (2m)^2$

$= \frac{1}{2} \cdot 18 \cdot 4 \text{ Nm}$

$= 36 \text{ Nm} = \boxed{36 j}$

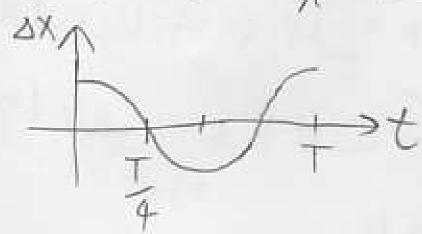
b) Cons. of E. $PE_i + \cancel{KE_i}^{\,0} = \cancel{PE_f}^{\,0} + KE_f$

$KE_{\Delta x = 0} = PE_{\Delta x = max} = \boxed{36 j}$

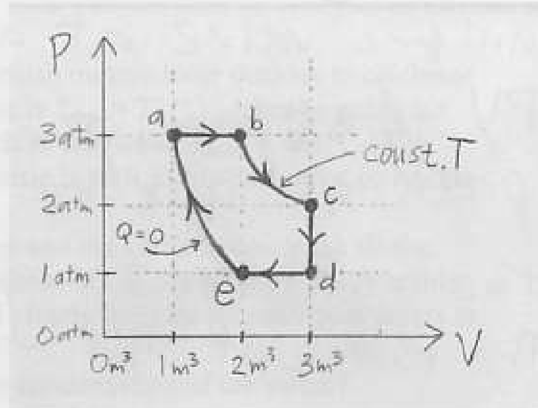
c) $KE_{\Delta x=0} = \frac{1}{2} m v^2_{\Delta x = 0} \Rightarrow v_{\Delta x=0} = \sqrt{\frac{2}{m} KE_{\Delta x = 0}} = \sqrt{\frac{2}{2kg} 36 j} = \sqrt{36 \frac{m^2}{s^2}} = \boxed{6 \text{ m/s}}$

d) mass on spring: $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{18 \frac{N}{m}}{2 kg}} = \sqrt{9 s^{-2}} = \boxed{3 s^{-1}}$

e) time to first cross $\Delta x = 0$: $\frac{T}{4} = \frac{1}{4}\frac{1}{f} = \frac{1}{4}\frac{2\pi}{\omega} = \frac{\pi}{2\omega} \approx \frac{3}{2 \cdot 3} s \approx \boxed{\frac{1}{2} s}$



Problem 5 (45 points) *PV* diagram of a peculiar heat engine. Consider a heat engine that undergoes the cycle shown in the *PV* diagram at the right. It consists of a (non-monoatomic) ideal gas that undergoes a constant pressure expansion from point *a* to point *b*, a constant temperature expansion from *b* to *c*, a constant volume path from *c* to *d*, a constant pressure compression from *d* to *e*, and an adiabatic ($Q_{in} = 0$) compression from *e* back to *a*. 1atm = 10^5 Pa = 10^5 N/m^2.



a) How much work is done *by* the gas along the path from point *a* to *b*? Please check your signs.

b) What is the change in energy in the gas along the path from point *b* to *c*? Explain your reasoning.

c) If $Q_{in} = 2.4 \times 10^5$ j along the path from point *b* to *c*, what is the work done *by* the gas from *b* to *c*?

d) How much work is done *by* the gas from point *c* to *d*?

e) How much work is done *by* the gas along the path *d*→*e*? $\times 10^5$

f) If the change in energy, $\Delta E_{e\to a}$, from point *e* to point *a* is 1.7 j, then how much work is done *by* the gas along this path?

g) How much net work is done *by* the engine during one full cycle (*a*→*b*→*c*→*d*→*e*→*a*)?

h) What is the net change in *entropy*, $\Delta S_{a\to e}$, in the gas along the following path: *a*→*b*→*c*→*d*→*e*? Please justify your answer.

a) $W_{on} = -\underbrace{P\Delta V}_{\text{area under path } a\to b} = -3\,atm \cdot (2-1)m^3 = -3\,atm\cdot m^3 = -3\times10^5\,Nm = -3\times10^5\,j$

$W_{by} = -W_{on} = \boxed{+3\times10^5\,j}$

b) Const. T ⇒ const E since $E = (\ )NkT$; so $\boxed{\Delta E_{b\to c} = 0}$

some constant

c) $\cancel{\Delta E_{b\to c}}^{\to 0} = Q_{in} + W_{on} \Rightarrow W_{by} = -W_{on} = +Q_{in} = \boxed{+2.4\times10^5\,j}$

d) $\Delta V = 0 \Rightarrow W_{by} = +P\cancel{\Delta V}^{\to 0} = \boxed{0\,j}$ (area under curve is zero)

e) $W_{by_{b\to e}} = +P\Delta V = 1atm\cdot(2-3)m^3 = -1\,atm\cdot m^3 = \boxed{-10^5\,j}$

0 adiabatic

f) $\Delta E_{e\to a} = \cancel{Q_{in}} + W_{on} = 1.7\times10^5\,j$ so $W_{by} = -W_{on} = -\Delta E = \boxed{-1.7\times10^5\,j}$

g) $W_{net_{a\to b\to c\to d\to e\to a,\ by}} = W_{a\to b,\ by} + W_{b\to c,\ by} + \ldots = (+3+2.4+0-1-1.7)\times10^5\,j$

$= \boxed{+2.7\times10^5\,j}$

please see back

h) entropy is a <u>state</u> variable, so it has a unique value at any particular point on a "PV" diagram. Therefore it doesn't matter what path we use to compute $\Delta S_{a\to e}$. I'll use the adiabatic path from $a \to e$ (which is just the reverse of the final path of the engine).

$Q_{in} = 0$ along this path, so for every infinitesimal step along the path: $dS = \frac{dQ_{in}}{T} = \frac{0}{T} = 0$

so $\boxed{\Delta S_{a\to e} = 0}$

(I wrote this for an infinitessimal step along the path since T changes along the path.)

Problem 6 (20 points) Tea time on the mountain

After a grueling climb to the top of Mount Dinali, an intrepid British mountaineer decides to celebrate with a cup of tea. At this high altitude, the boiling point of water is $T_{boil} = 77$ °C. After he pours his tea, the mountaineer places the kettle directly on the snow, which is very cold, $T_{snow} = -23$ °C. The kettle initially contains 2 kg of steam at 77 °C. The top of the kettle is well insulated so that no heat is exchanged between the kettle and the air.

a) How much heat, Q, must be exchanged between the snow and the kettle to condense all the steam in the kettle into liquid water at 77 °C? The latent heat of vaporization for water at this altitude is $L_{vapor} \approx 2 \times 10^6$ j/kg. Take care with signs and clearly indicate whether heat enters or leaves the kettle.

b) What is the change in entropy inside the kettle due to the condensation of the steam?

c) What is the change in entropy of the snow during this process?

d) What is the change in entropy in the universe as a result of this process? Please indicate whether this is positive, negative, or zero.

a) $Q = m_{H_2O} L_{vapor} = 2\,kg \cdot 2\times10^6\,j/kg = \boxed{4\times10^6\,j}$ ← heat leaves the kettle (so $Q_{in\,kettle} < 0$)

b) $\Delta S_{kettle} = \frac{Q_{in\,k}}{T_{kettle}} = \frac{-4\times10^6\,j}{350K} \cong \boxed{-11{,}000\,j/K}$

convert to kelvin: $T_{kettle} = (77 + 273)K = 350\,K$

c) $\Delta S_{snow} = \frac{Q_{in\,snow}}{T_{snow}} = \frac{+4\times10^6\,j}{250K} \cong \boxed{+16000\,j/K}$ $Q_{in\,snow} = -Q_{in\,kettle}$

$T_{snow} = (-23 + 273)K = 250\,K$

d) $\Delta S_{universe} = \Delta S_{snow} + \Delta S_{kettle}$

$\cong +16{,}000\,\frac{j}{K} - 11{,}000\,j/K$

$\cong \boxed{+5{,}000\,j/K}$

114
35 ⟌ 400000
35
50
35
150

16000
25 ⟌ 400000
25
150