Physics 8A Section 2 Final

SOLUTIONS

name

May 21, 2003

SID Number

5 pm – 8 pm

discussion session number / GSI name

a) DON'T OPEN THIS EXAM UNTIL INSTRUCTED TO BEGIN
b) Sit one seat away from anyone else.
c) Do all your work on the page (front/back) indicated for each problem.
d) Show all work; don't just write an answer without showing your reasoning.
e) This is a closed book exam but calculators and one sheet of notesare allowed.
f) To simplify the math, take the acceleration due to gravity as g = 10 m/s^2.
g) There are **NINE** problems on the exam.
h) Possibly useful equations are shown on the next page.

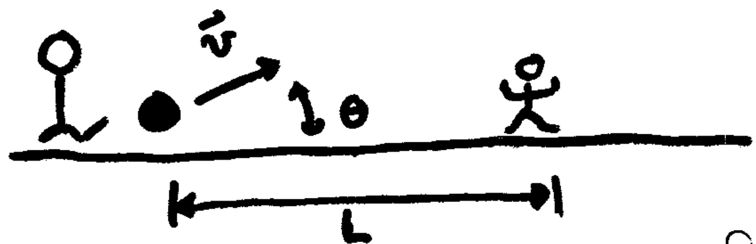
1	5
2	10
3	15
4	15
5	5
6	10
7	10
8	10
9	10
total	90

$F = dp / dt = m a$
$p = mv$
$F_c = m v^2 / r$
$x = x_0 + v_0 t + 1/2 a t^2$
$F_f = \mu_f N$
$W = F x$
$U = m g h$
$K = 1/2 m v^2$
$I = mr^2$
$I = 1/2 mr^2$
$I = I_{com} + mh^2$
$\tau = I \alpha$
$\tau = r \times F$
$\tau = dL/dt$
$L = I \omega$
$v = \omega r$
$a = \alpha r$
$K = 1/2 I \omega^2$
$v_1 A_1 = v_2 A_2$
$p + 1/2 \rho v^2 + \rho g y = \text{constant}$
$\omega = (k/m)^{1/2}$
$T = 2 \pi (L/g)^{1/2}$
$\Delta E = Q - W$
$\Delta E = n C_V \Delta T$
heat of fusion of H_2O = 333 kJ/kg
specific heat of ice = 2220 J/kg K
$\rho (H_2O) = 1000 \text{ kg/m}^3$
$P = k A (T_1 - T_2) / L$
$P = \sigma \varepsilon A (T_1^4 - T_2^4)$
$\sigma = 5.7 \times 10^{-8} \text{ W/m}^2\text{/K}^4$
$pV = n R T$
$C_V = 3/2 R$
$C_P = C_V + R$

$R = 8.3 \text{ J / mol}$
$\gamma = C_P / C_V$
$\Delta S = \Delta Q / T$
$pV^\gamma = \text{constant}$ - adiabatic
$W = n R T \ln (V_f / V_i)$ - isothermal
$\varepsilon = W / Q$
$\Delta S = nR \ln (V_f / V_i) + nC_V \ln (T_f / T$

1) A soccer ball is kicked from the ground with an initial speed v = 20 m/s at an upward angle θ = 45 degrees. A second player at a distance L =50 m away in the direction of the kick, starts running at the instant of the kick, and meets the ball as it hits the ground.
(5 points)

(a) What was the average speed and direction of the second player?



The position of the ball as a function of time is

$$x(t) = V_{ox} t$$
$$y(t) = V_{oy} t - \frac{1}{2} g t^2$$

where

$$V_{ox} = (20 m/s) \cos 45^\circ = 14.1 \ m/s$$
$$V_{oy} = (20 m/s) \sin 45^\circ = 14.1 \ m/s$$

The time t^* when the ball hits the ground is:

$$y(t^*) = 0$$
$$V_{oy} t^* - \frac{1}{2} g t^{*2} = 0$$
$$t^* = 2 V_{oy} / g = \frac{2 \cdot (14.1 m/s)}{10 \ m/s^2} = 2.83 \text{ seconds}$$

The horizontal position when the ball hits the ground is

$$x(t^*) = V_{ox} t^* = (14.1 m/s)(2.83 s)$$
$$= 40 m$$

Because the 2nd player starts 50 m away, he must run 10 m toward the 1st player in the 2.83 seconds it takes the ball to get there. So his average speed is

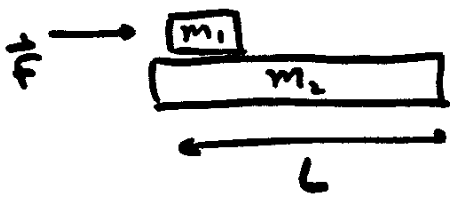
$$V_{avg} = \frac{\Delta X}{\Delta t} = \frac{10m}{2.83\,s} = \boxed{3.5\ m/s}$$

2) A small block with mass $m_1 = 2$ kg rest on the left edge of a second block of mass $m_2 = 8$ kg and length L = 3 m. A coefficient of kinetic friction $\mu = 0.3$ exists between the two blocks. The surface on which the large block sits is frictionless. A constant horizontal force F = 10 N is applied to block m_1, setting it and the larger block in motion to the right.

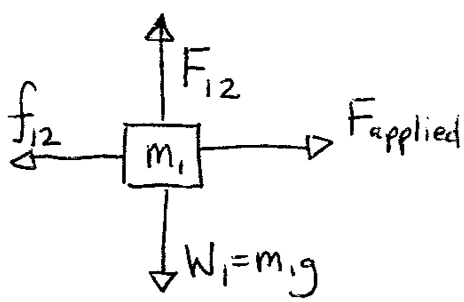
(10 points)

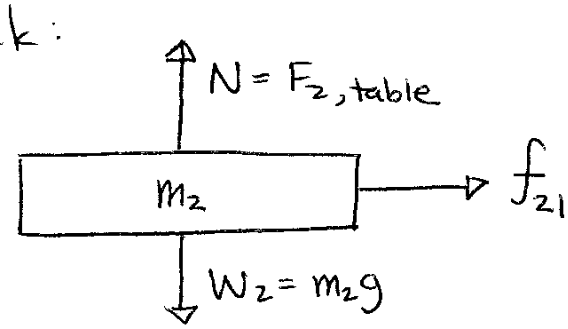
(a) How long does it take for block m_1 to reach the right edge of block m_2?

(b) How far does block m_2 move in the process?



(a) F.B.D for each block:





Newton's 3rd law:

- $F_{net,y} = 0 \rightarrow m_1 g = F_{12}$
- $F_{net,x} = m_1 a_1 = F_{applied} - f_{12}$

$$= F_{applied} - \mu F_{12}$$
$$= F_{applied} - \mu m_1 g$$

Newton's 3rd law:

- $F_{net,y} = 0 \rightarrow m_2 g = N$
- $F_{net,x} = m_2 a_2 = f_{12}$

$$= \mu m_1 g$$

So the acceleration of the blocks is

$$a_1 = \frac{F_{app}}{m_1} - \mu g$$
$$= \frac{10\,N}{2\,kg} - (.3)(10\,m/s^2)$$
$$= 2\,m/s^2$$

$$a_2 = \frac{\mu m_1 g}{m_2} = \frac{(.3)(2\,kg)(10\,m/s^2)}{(8\,kg)}$$
$$= .075\,m/s^2$$

The position of each block as a function of time is

$$x_1(t) = \frac{1}{2} a_1 t^2$$
$$x_2(t) = \frac{1}{2} a_2 t^2$$

where x_2 is the position of the left side of block 2. When the relative position between $x_1(t)$ and $x_2(t)$ equals L, the top mass has reached the right side of the bottom mass:

$$x_1(t^*) - x_2(t^*) = L = \frac{1}{2}(a_1 - a_2)\, t^{*2}$$

$$t^* = \sqrt{\frac{2L}{a_1 - a_2}} = \sqrt{\frac{2(3\,m)}{2\,m/s^2 - .075\,m/s^2}} = \boxed{1.77 \text{ sec}}$$

(b) During this 1.77 seconds, the second block moves

$$x_2(t^*) = \frac{1}{2} a_2 t^{*2} = \frac{1}{2}(.075\,m/s^2)(1.77\,sec)^2$$
$$= \boxed{.12\,m}$$

3) Two masses are placed on a frictionless track. Mass $m_1 = 1$ kg is given an initial velocity v = 3 m/s to the right. It collides elastically with mass $m_2 = 2$ kg with was initially at rest. Mass m_2 runs into a massless spring that has a force constant k = 50 N/m and is attached to a wall at the far right.
(15 points)

(a) What is the velocity of each mass just after the collision?
(b) What is the maximum compression of the spring?
(c) How long is mass m_2 in contact with the spring?

(a) before the collision

$P_i = m_1 V$

$E_i = \frac{1}{2} m_1 V^2$

after the collision

$P_f = m_2 V_2 - m_1 V_1$

$E_f = \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_1 V_1^2$

Both momentum and energy are conserved during an elastic collision. Conservation of p gives

$$m_1 V = m_2 V_2 - m_1 V_1 \quad \rightarrow \quad V_2 = \frac{m_1}{m_2}(V + V_1)$$

Conservation of E gives

$$\frac{1}{2} m_1 V^2 = \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_1 V_1^2$$

$$= \frac{1}{2} m_2 \left(\frac{m_1}{m_2}(V+V_1) \right)^2 + \frac{1}{2} m_1 V_1^2$$

With some algebra this becomes

$$0 = \left[\frac{1}{2} \frac{m_1^2}{m_2} + \frac{1}{2} m_1 \right] V_1^2 + \left[\frac{m_1^2}{m_2} V \right] V_1 + \left[\frac{1}{2} \frac{m_1^2}{m_2} V^2 - \frac{1}{2} m_1 V^2 \right]$$

$$= (.75 \text{ kg}) V_1^2 + (1.5 \text{ kg} \tfrac{m}{s}) V_1 + (-2.25 \text{ kg} \tfrac{m^2}{s^2})$$

5

Using the quadratic formula

$$V_1 = \frac{-1.5 \text{ kg}\frac{m}{s} \pm \sqrt{\left(1.5 \text{ kg}\frac{m}{s}\right)^2 - 4(.75 \text{ kg})\left(-2.25 \text{ kg}\frac{m^2}{s^2}\right)}}{(2)(.75 \text{ kg})}$$

The solutions to this are $V_1 = -3$ m/s (which is the initial setup, before the collision) and $\boxed{V_1 = +1 \text{ m/s}}$ which must be the speed of mass m_1 after the collision. The + sign, as indicated in the picture, means its moving $\boxed{\text{left.}}$ From the cons. of momentum equation gives

$$V_2 = \frac{m_1}{m_2}(V+V_1) = \boxed{+2 \text{ m/s}}$$

Here, the + sign means mass 2 is moving $\boxed{\text{right}}$.

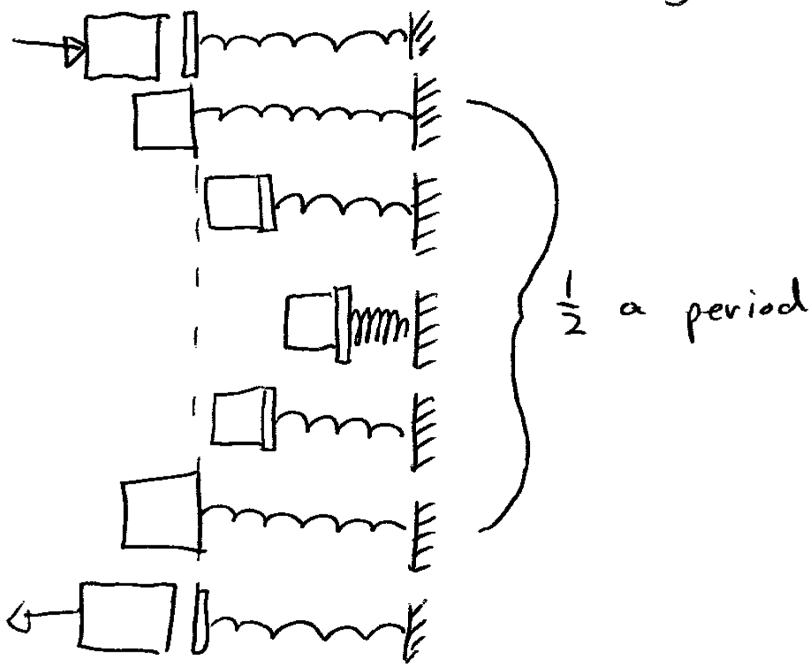
(b) Only mass 2 hits the spring. Conservation of energy before it hits, and after it's maximally compressed gives

$$\frac{1}{2} m_2 V_2^2 = \frac{1}{2} k x^2$$

$$\to x = \sqrt{\frac{m_2}{k}} \cdot V_2 = \sqrt{\frac{2 \text{ kg}}{50 \text{ kg}/s^2}} (2 \text{ m/s})$$

$$= \boxed{.4 \text{ m}}$$

(c) The time it takes m_2 to completely compress the spring <u>and</u> get pushed back out is $\frac{1}{2}$ the period at which the spring / mass system would oscillate at if they were glued together:

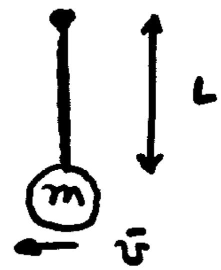


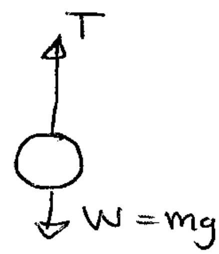
$$\Delta t = \frac{T}{2} = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{1}{2} \times \frac{2\pi}{\sqrt{k/m_2}} = \frac{1}{2}\,\frac{2\pi}{\sqrt{\frac{50\ \mathrm{kg/s^2}}{2\ \mathrm{kg}}}}$$

$$\cong \boxed{.63\ \mathrm{sec}}$$

4) A mass m = 0.5 kg hangs from a stiff, massless cable of length L = 1 m. As it passes through the lowest part of its path it has a speed v = 3 m/s.
(15 points)

(a) What is the tension in the cable at this lowest part of the masses motion?
(b) When the mass reaches its highest point, what angle does the cable make with the vertical?
(c) What is the tension in the cable when the mass reaches its highest point?





(a) FBD at lowest point:

Newton's 3rd law in vertical direction:

$$F = ma_v = T - mg \rightarrow T = m(a_v + g)$$

Because the mass is moving in a circle, we must have the vertical acceleration equal to central acceleration:

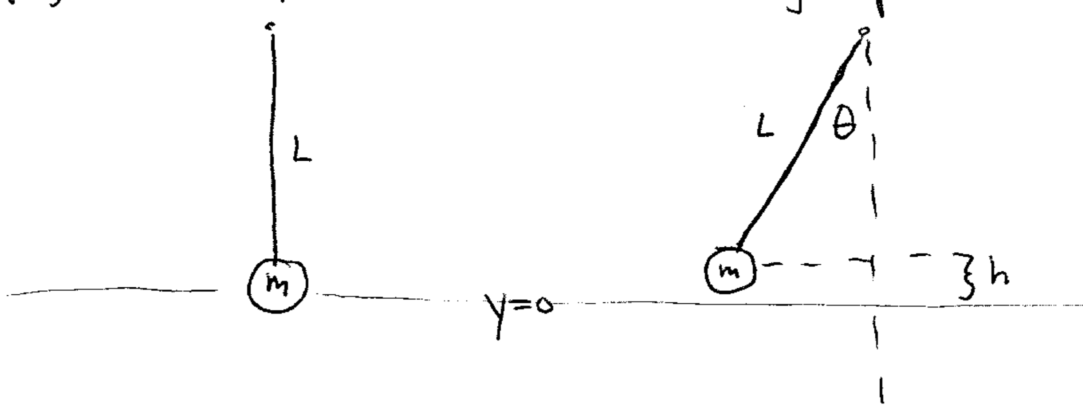
$$a_v = a_c = v^2/r$$

so

$$T = m\left(\frac{v^2}{r} + g\right) = (.5\,\text{kg})\left(\frac{(3\,\text{m/s})^2}{1\,\text{m}} + 10\,\text{m/s}\right)$$

$$= \boxed{9.5\ \text{N}}$$

(b) low point high point



At the low point:

$$E = T = \frac{1}{2}mv^2$$

At the high point

$$E = U = mgh = mg\,(L - L\cos\theta)$$

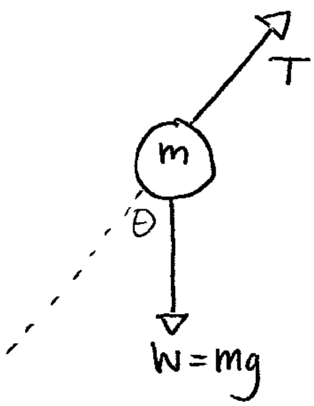
Cons. of energy gives

$$\frac{1}{2}mv^2 = mg\,L\,(1-\cos\theta)$$

$$\cos\theta = 1 - \frac{v^2}{2gL} = .55$$

$$\theta = .9884^{R} = \boxed{56.6^\circ}$$

(c) FBD at the highest point



Newton's law in the radial direction:

$$ma_r = T - W\cos\theta$$

Because the mass is moving in a circle its radial acceleration is always central acceleration:

$$a_r = a_c = v^2/r$$

At the high point, $v=0$ so $a_r=0$. This gives

$$T = W\cos\theta$$

$$= mg\cos 56.6$$

$$= (.5\text{ kg})(10\text{ m/s}^2)(.55) = \boxed{2.75\text{ N}}$$

5) A horizontal pipe has water flowing in it to the right at a rate of 4×10^{-4} m^3/s. In a wide-diameter part of the pipe (with radius $r_1 = 2.5$ cm) a vertical tube supports a column of water of height $h_1 = 10$ cm above the pipe. In a narrow-diameter part of the pipe (with radius r_2) a vertical tube supports a column of water of height $h_2 = 5$ cm above the pipe.
(5 points)

(a) What is the radius r_2?

Call P_1 the pressure at the top edge of the thick pipe, where the water is moving at speed V_1 at height r_1 (measured relative to the pipe axis).

Call P_2 the pressure at the top edge of the thin pipe, where the water is moving at speed V_2 at height r_2.

Bernoulli's equation gives

$$P_1 + \rho g r_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g r_2 + \tfrac{1}{2}\rho v_2^2 \quad (1)$$

The continuity equation is

$$Q = V_1 A_1 = V_2 A_2 \quad (2)$$
$$= \pi V_1 r_1^2 = \pi V_2 r_2^2$$

The pressure at the top of the 1st tube is atmospheric pressure, P_0. At the bottom of the tube the pressure is $P_0 + \rho g h_1$. This must be the same as the pressure of the water at the top of the thick tube, so

$$P_0 + \rho g h_1 = P_1 \qquad (3)$$

Similarly, for tube #2:

$$P_0 + \rho g h_2 = P_2 \qquad (4)$$

Combining equations (1), (2), (3) & (4) gives

$$\cancel{P_0} + \rho g h_1 + \rho g r_1 + \frac{1}{2}\rho\left(\frac{Q}{\pi r_1^2}\right)^2$$

$$= \cancel{P_0} + \rho g h_2 + \rho g r_2 + \left(\frac{Q}{\pi r_2^2}\right)^2 \cdot \frac{1}{2}\rho$$

With the numbers given in the problem for

$Q = 4 \cdot 10^{-4}\ m^3/s$

$r_1 = .025\ m$

$h_1 = .1\ m$

$h_2 = .05\ m$

$\rho = 1000\ kg/m^3$

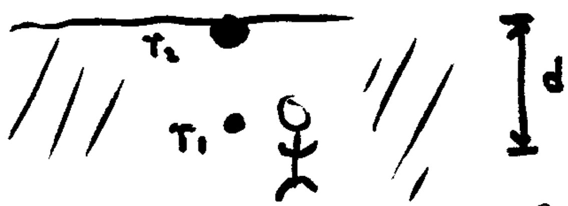
$g = 10\ m/s^2$

The answer for r_2 is:

$$\boxed{r_2 = .0105\ m}$$

6) A diver burps an air bubble of radius $r_1 = 2$ mm at some depth d. The bubble rises and as it reaches the surface it has a radius $r_2 = 3$ mm. Assume that the temperature of the bubble remains constant.
(10 points)

(a) What was the depth d of the diver?
(b) What was the absolute pressure at this depth?



(a) Water pressure as a function of depth is

$$P = P_0 + \rho_w g d$$

where P_0 = atmospheric pressure is the pressure above the water. The gas inside the bubble is at the same pressure as the surrounding water. The ideal gas law for each bubble gives:

$$nRT = P_0 V_2 \quad \text{at surface}$$

$$nRT = (P_0 + \rho_w g d) V_1 \quad \text{at depth } d$$

Because the temperatures are the same:

$$P_0 V_2 = (P_0 + \rho_w g d) V_1$$

$$\rightarrow \quad d = \frac{P_0}{\rho_w g}\left(\frac{V_2}{V_1} - 1\right) = \frac{P_0}{\rho_w g}\left(\frac{r_2^3}{r_1^3} - 1\right)$$

$$= \frac{10^5 \text{ Pa}}{(1000 \text{ kg/m}^3)(10 \text{ m/s}^2)}\left(\frac{(3\text{mm})^3}{(2\text{mm})^3} - 1\right) = \boxed{23.75 \text{ m}}$$

8

(b) $P = P_0 + \rho_w g d$

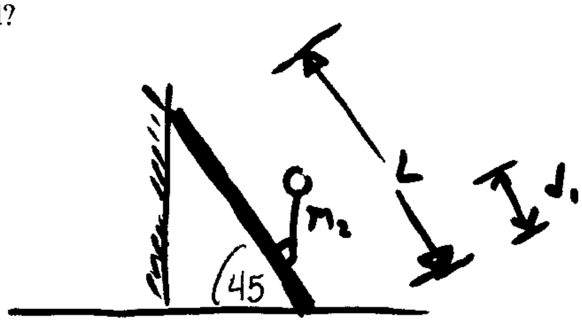
$$= 10^5\ Pa + (1000\ kg/m^3)(10\ m/s^2)(23.75\ m)$$

$$= 337.5\ kPa$$

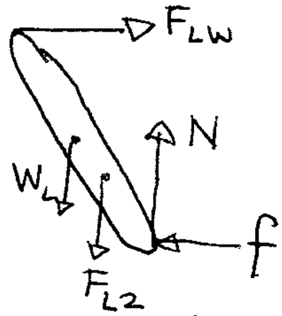
$$= \boxed{3.375\ atm}$$

7) A ladder of length L = 15m and mass m_1 = 50 kg rests against a wall at an angle of 45 degrees with respect to the horizontal. A man of mass m_2 = 80 kg stands on the ladder at a distance d_1 = 4 m from the bottom of the ladder.
(10 points)

(a) What are the horizontal and vertical forces on the bottom of the ladder?
(b) If the ladder is on the verge of slipping when the person is at a distance d_2 = 9 m from the bottom, what is the coefficient of friction between the ladder and the ground?



(a) Assume the wall is frictionless. Then the F.B.D. for the ladder is



F_{LW} = force on ladder from wall
W_L = weight of ladder $= m_1 g$
F_{L2} = force on ladder from $m_2 = m_2 g$
N = normal force from ground
f = friction " $\leq \mu_s N$

Newton's 3rd law in the y direction:

$$F_{net,y} = 0 = N - W_L - F_{L2}$$

$$\Rightarrow N = W_L + F_{L2} = (m_1+m_2)g$$

$$\boxed{= 1300\ N}$$

Newton's 3rd law in the x direction:

$$F_{net\ x} = 0 = F_{LW} - f \Rightarrow f = F_{LW}$$

To determine F_{LW} consider the torques about the bottom of the ladder:

$$\tau = I\alpha = 0 = L\,(F_{LW}\sin 45) - \frac{L}{2}(W_L\cos 45) - d_1\,(F_{L2}\cos 45)$$

$$\hookrightarrow 0 = L\cdot F_{LW}\tan 45 - \frac{L}{2}\cdot m_1 g - d_1\cdot m_2 g$$

$$F_{LW} = \left(\frac{L}{2}m_1 + d_1 m_2\right)g \,/\, L\cdot\tan 45$$

$$= \frac{\left(\frac{15\,m}{2}\cdot 50\,kg + 4m * 80\,kg\right)10\,m/s^2}{15\,m * 1}$$

$$= 463\ N$$

so $f = F_{LW} = \boxed{463\ N}$

(b) Using $d_2 = 9m$ instead of $d_1 = 4m$ in the above equation:

$$F_{LW} = f_{max} = \frac{\left(\frac{15\,m}{2}\cdot 50\,kg + * 9m * 80\,kg\right)10\,m/s^2}{15m * 1}$$

$$= 730\ N$$

The vertical force on the bottom is still

$$N = 1300\ N$$

Using the equation for maximum static friction

$$f_{max} = \mu_s N \rightarrow \mu_s = \frac{f_{max}}{N} = \boxed{.56}$$

8) A cup of water at 0 °C is insulated from its environment. The cup has an open diameter d = 4 cm and faces the night sky, which is at a negligibly cold temperature. Assume an emissivity ε = 0.5 for water.
(10 points)

(a) What is the power loss of the water?
(b) How long does it take for 10 cc of water in this cup to freeze?

(a) $P = \sigma \varepsilon A (T_1^4 - T_2^4)$

$A = \pi r^2 = \pi (2\,cm)^2 = .001257\ m^2$

$T_1 =$ water temp $= 0°C = 273\ K$

$T_2 =$ sky temp $\cong 0\,K$

plugging in numbers:

$\boxed{P = .2\,W}$

(b) 10 cc of water weighs

$m = 10\ cm^3 \times \left(\frac{m}{100\,cm}\right)^3 \times \frac{1000\ kg}{m^3} = .01\ kg$

To freeze this much water requires removing heat:

$\Delta Q = L m = (333\ kJ/kg)(.01\ kg)$

$= 3.33\ kJ = 3330\ J$

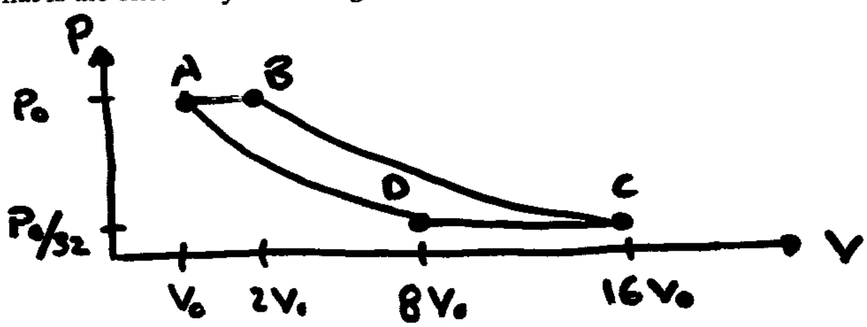
The time it takes to loose this heat is:

$P = \frac{\Delta Q}{\Delta t} \rightarrow \Delta t = \frac{\Delta Q}{P} = \frac{3330\ J}{.2\ W} = 16650\ sec$

$= \boxed{4.6\ hours}$

9) One mole of n ideal gas is the working substance of an engine that operates on the following cycle:
A to B (isobaric at P_0, V_0 to $2V_0$)
B to C (adiabatic, P_0 to P_0 /32, 2V to $16V_0$)
C to D (isobaric at P_0 /32, $16V_0$ to $8V_0$)
D to A (adiabatic, P_0 /32 to P_0 , $8V_0$ to V_0)
(10 points)

(a) Is the gas monoatomic, diatomic, or polyatomic?
(b) What is the efficiency of the engine?



(a) During the adiabatic expansion from B → C we have

$$P_B V_B^\gamma = P_C V_C^\gamma \rightarrow P_0 \cdot (2V_0)^\gamma = \frac{P_0}{32} \cdot (16V_0)^\gamma$$

so

$$2^\gamma = \frac{16^\gamma}{32}, \text{ or } 8^\gamma = 32$$

the solution to this is $\gamma = 5/3$ so the gas must be $\boxed{\text{mono atomic}}$

(b) Q_{in} = heat that flows in during A → B

$$= n C_p \Delta T = n \cdot \frac{5}{2} R \cdot (T_B - T_A)$$

$$= \frac{5}{2} (P_B V_B - P_A V_A) = \frac{5}{2} (2 P_0 V_0 - P_0 V_0) = \frac{5}{2} P_0 V_0$$

Q_{out} = heat that flows out during C → D

$$= |n C_p \Delta T| = n \cdot \frac{5}{2} R (T_C - T_D)$$

$$= \frac{5}{2} (P_C V_C - P_D V_D) = \frac{5}{2} (\frac{1}{2} P_0 V_0 - \frac{1}{4} P_0 V_0) = \frac{5}{8} P_0 V_0$$

11

So the efficiency is

$$\varepsilon = \frac{\text{what you get out}}{\text{what you put in}} = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

$$= 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{5/8 \; P_0 V_0}{5/2 \; P_0 V_0}$$

$$\boxed{= \frac{3}{4}}$$