Physics 8A (sec. 1) Midterm Exam #2 April 15, 2002

You may use one (1) card, not larger than 3"x5", as a memory aid, but no other papers, and no books. The exam totals 170 points.

Constants: $G = 6.67\times10^{-11}\ Nm^2(kg)^{-2}$; $g = 9.8\ m\,sec^{-2}$

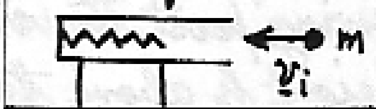
(15)(1) A thin rod of mass m and length l can rotate about an axis through one end of the rod. The rod is pulled away from the vertical and allowed to swing (from rest) like a pendulum, passing through the vertical position with angular speed ω. (a) Calculate the angular momentum L of the rod when it is vertical; (b) Calculate the kinetic energy K of the rod when it is vertical; (c) The rod continues to swing past the vertical until the center of mass reaches a height h above the lowest position of the center of mass. Neglecting friction and air resistance, calculate the value of h using energy considerations. [a = b = c = 5 points]

(15)(2) An astronomical object called a neutron star has a radius of 10 km, but a mass equal to that of the Sun (2×10^{30} kg). (a) Calculate the acceleration g' due to gravity at a point on the surface of the neutron star; (b) Assuming that the star does not rotate, calculate the speed of an object, dropped from a height of 1.0 meter from the surface of the neutron star, when the object reaches the surface. [a = 8, b = 7 pts]

(continued →)

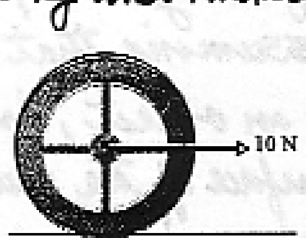
(15) (3) At a particular instant of time, a particle of mass 3.0 kg has a velocity of 6.0 m/sec in the direction of decreasing y and a particle of mass 4.0 kg has a velocity of 7.0 m/sec in the direction of increasing x. (a) Using unit vectors, calculate the velocity vector $\underline{v}_{cm}$ of the center of mass of the system; (b) Calculate the magnitude of $\underline{v}_{cm}$; (c) Calculate the angle θ between $\underline{v}_{cm}$ and the x-axis. [a=b=c= 5 points each]

(20) (4) A ball of mass m is projected horizontally into a "spring-barrel" (of mass M) with speed v_i. The "spring-barrel" is initially at rest on a frictionless horizontal surface, as shown.



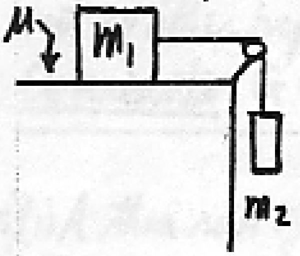
The ball compresses the spring and remains in the barrel at the point of maximum compression of the spring. No mechanical energy is dissipated by friction. (a) Calculate the speed v of the barrel after the spring is compressed; (b) Calculate the fraction f of the initial kinetic energy of the ball that is stored in the spring (a=b=10 pts)

(25) (5) A constant horizontal force of 10 N. is applied to a wheel of mass 10 kg and radius 0.30 meter, as shown. The wheel is NOT a uniform cylinder. The wheel rolls without slipping on the horizontal surface shown, and the acceleration of its center of mass is 0.60 m/sec^2. (a) Calculate the magnitude and specify the direction of the frictional force exerted on the wheel; (b) Calculate the rotational inertia (moment of inertia) of the wheel relative to an axis normal to the plane of the wheel and through its center of mass. [a=10, b=15]



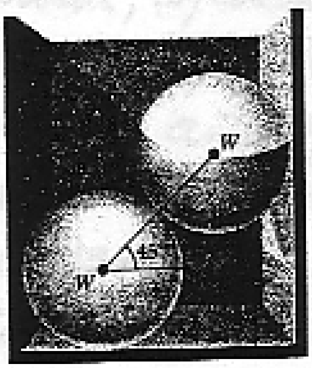
(continued →)

(40) (6) Two blocks are connected by a massless and inextensible string as shown, and are released from rest. The coefficient of



kinetic friction between block m_1 and the surface is μ. The pulley is massless and frictionless. Using energy considerations, calculate the common speed v of the two blocks after they have moved a distance L.

(40) (7) Two identical uniform frictionless spheres, each of weight W, rest in a rigid rectangular container, as shown. The line



of centers of the spheres makes an angle of 45° with the horizontal. Calculate, in terms of W, the forces acting on the spheres due to the container and due to one another.

Physics 8A Midterm #2 Solutions

(1)(a) The angular momentum L is given by

$$L = I\omega$$

where I is the moment of inertia of the rod with respect to an axis through its pivot. For a rod of mass m and length l rotating about an axis through one end, $I = \frac{1}{3}ml^2$, so

$$\boxed{L = \frac{1}{3}ml^2\omega}$$

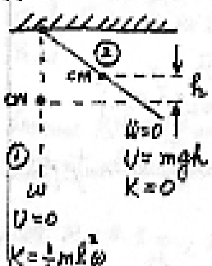
(b) The kinetic energy K in this situation is the kinetic energy of rotation, so

$$K = \frac{1}{2}I\omega^2$$

and, with $I = \frac{1}{3}ml^2$,

$$\boxed{K = \frac{1}{6}ml^2\omega^2}$$

(c) Here the situation is as shown. In the vertical position, the angular speed is ω, the kinetic energy $K = (1/6)ml^2\omega^2$ and we define the gravitational potential energy $U \equiv 0$. In the second position ($\omega = 0$) so $K = 0$ and $U = mgh$. Since there is no friction or air resistance the total energy $E = K + U$ is conserved, so



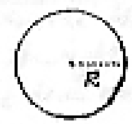
$$U_1 + K_1 = K_2 + U_2$$

leading to

$$0 + \frac{1}{6}ml^2\omega^2 = 0 + mgh$$

$$\boxed{h = (l^2\omega^2/6g)}$$ is the required answer for h

(2)(a) The neutron star exerts a force F_1 on mass m, where



$$F_1 = G\frac{mM_N}{R^2}$$

where M_N is the mass of the neutron star. The force F_1 is also given by

$$F_1 = mg'$$

where g' is the acceleration due to gravity at a point on the surface of the neutron star. Then

$$mg' = (GmM_N/R^2)$$

$$g' = (GM_N/R^2)$$

With $G = 6.67 \times 10^{-11}\ Nm^2(kg)^{-2}$, $M_N = 2 \times 10^{30}\ kg$, $R = 10\ km = 10^4\ m$,

$$\boxed{g' = 1.33 \times 10^{12}\ m/sec^2}$$

(b) If an object is dropped from rest (initial speed $v_0 = 0$) from a height $h = 1$ meter from the surface of the neutron star, its speed v when it reaches the surface of the star is given by

$$v^2 = v_0^2 + 2g'h$$

$$v^2 = 0 + 2g'h$$

$$v = (2g'h)^{1/2}$$

With $g' = 1.33 \times 10^{12}\ m/sec^2$, $v = [(2)(1.33 \times 10^{12})(1)]^{1/2}\ m/sec$

$$\boxed{v = 1.63 \times 10^6\ m/sec}$$

(3)(a) Using unit vector notation and calling the particles "1" and "2", we have:

$$m_1 = 3\ \text{kg},\ \underline{v}_1 = -6\hat{j}$$
$$m_2 = 4\ \text{kg},\ \underline{v}_2 = 7\hat{i}$$

Then

$$\underline{x}_{cm} = \frac{m_1\underline{x}_1 + m_2\underline{x}_2}{(m_1+m_2)} \Longrightarrow \underline{v}_{cm} = \frac{d}{dt}[\underline{x}_{cm}] = \frac{m_1\underline{v}_1 + m_2\underline{v}_2}{(m_1+m_2)}$$

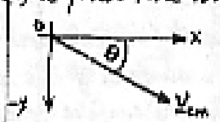
so

$$\underline{v}_{cm} = \frac{1}{7}\left[28\hat{i} - 18\hat{j}\right] \Longrightarrow \boxed{\underline{v}_{cm} = (4\hat{i} - 2.57\hat{j})}$$

(b) The speed $|\underline{v}_{cm}| = (\underline{v}_{cm}\cdot\underline{v}_{cm})^{1/2} = (16 + 6.61)^{1/2} = (22.61)^{1/2}$

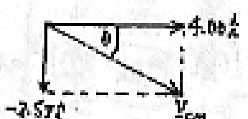
$$\boxed{|\underline{v}_{cm}| = 4.75\ \text{m/sec}}$$

(c) To find the angle θ between $\underline{v}_{cm}$ and the x-axis, we have



$$(\underline{v}_{cm}\cdot\hat{i}) = |\underline{v}_{cm}||\hat{i}|\cos\theta$$
$$4 = (4.76)(1)\cos\theta$$
$$0.841 = \cos\theta$$
$$\boxed{\theta = 32.7^\circ}$$

This answer can also be obtained by a graphical method:



$$\tan\theta = \frac{2.57}{4.00} = 0.6425$$
$$\boxed{\theta = 32.7^\circ}$$

(4)(a) Linear momentum of the (barrel + ball) is conserved so

$$mv_i + 0 = (m+M)v$$

where M is the mass of the barrel and v the final speed. Then

$$\boxed{v = \frac{mv_i}{(M+m)}}$$

(b) If K_i is the initial KE of (barrel + ball), U_i is the initial elastic PE of the spring, K_f is the final KE of (barrel + ball), U_f the final elastic PE of spring, then conservation of energy gives

$$K_i + U_i = K_f + U_f$$
$$(K_i - K_f) = (U_f - U_i) = \Delta U$$

where ΔU is the change in the elastic PE of the spring. With

$$K_i = \tfrac{1}{2}mv_i^2 \quad ; \quad K_f = \tfrac{1}{2}(m+M)v^2$$

we get $\Delta U = \frac{1}{2}mv_i^2 - \frac{1}{2}(m+M)v^2$, where $v = (mv_i)/(m+M)$, so

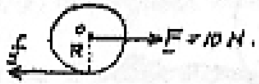
$$\Delta U = \tfrac{1}{2}mv_i^2 - \tfrac{1}{2}(m+M)\frac{m^2v_i^2}{(M+m)^2}$$
$$\Delta U = \tfrac{1}{2}mv_i^2\left[1 - \frac{m}{(M+m)}\right] = \tfrac{1}{2}mv_i^2\left(\frac{M}{m+M}\right)$$

where ΔU is the elastic PE stored in the spring (since $U_i = 0$) and the required fraction f is given by

$$f = \frac{\Delta U}{K_i} = \left(\frac{2}{mv_i^2}\right)\left(\frac{1}{2}mv_i^2\right)\left(\frac{M}{M+m}\right) = \frac{M}{M+m}$$

$$\boxed{f = \frac{M}{M+m}}$$

(5)(a) The frictional force $\underline{f}$ exerted on the wheel by the surface is tangent to the circumference of the wheel and is directed <u>opposite</u> to the applied force



$|\underline{F}| = 10$ Newtons. <u>To calculate</u> f, we have Newton's Second Law,

$$F_{net} = F - f = ma_{cm} \implies f = F - ma_{cm}$$

where m is the mass of the wheel and a_{cm} is the acceleration of the cm. Then

$$f = 10 - (10)(0.6)\ \text{N.} \implies \boxed{f = 4.0 \text{ Newtons}}$$

(b) <u>To calculate</u> the rotational inertia (moment of inertia) I about the axis normal to the plane of the wheel and through its center O, we use Newton's Second Law

$$\tau = I\alpha$$

where τ is the torque exerted on the wheel and α is the angular acceleration of the wheel. Since $\underline{F}$ acts through the center of the wheel, $\underline{F}$ exerts no torque (since its lever arm relative to O is zero). Thus

$$\tau = Rf = I\alpha = I\,(a_{cm}/R)$$

because $\underline{R}$ and $\underline{f}$ are mutually perpendicular. Since the wheel rolls without slipping,

$$v_{cm} = R\omega \implies a_{cm} = R\alpha \implies \alpha = (a_{cm}/R)$$

With $R = 0.3$ m., $f = 4$ N., $a_{cm} = 0.6$ m/sec^2, we have

$$(0.3)(4) = I\left(\frac{0.6}{0.3}\right) \implies 1.2 = 2I \implies \boxed{I = 0.6 \text{ kg m}^2}$$

(This result confirms that the wheel is <u>not</u> a uniform cylinder, whose $I = \frac{1}{2}mR^2 = \frac{1}{2}(10)(0.3)^2 = 0.45$ kg m^2)

(6) <u>Initially</u>, the kinetic energy K of each block is zero: $K_1 = K_2 = 0$. When the blocks move a distance L (m_1 moves horizontally to the right and m_2 moves vertically downward), the work W_2 done on m_2 by the gravitational force is

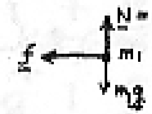
$$W_2 = m_2 g L$$

(since the force $|m_2 g|$ is parallel to the displacement L) and the work W_1 done on m_1 by the frictional force $\underline{f}$ is

$$W_1 = -fL$$

(since the frictional force is antiparallel to the displacement of mass m_1). Since m_1 does not move vertically, its force diagram is

$\underline{N}$ = normal force exerted on m_1 by surface



$$\implies N = m_1 g \implies f = \mu N = \mu m_1 g \quad \text{leading to}$$

$$W_1 = -\mu m_1 g L$$

The total work W done on the system $(m_1 + m_2)$ is

$$W = W_1 + W_2 = -\mu m_1 g L + m_2 g L = gL\,(m_2 - \mu m_1)$$

<u>From the work-kinetic energy theorem</u>, $W = \Delta K = K_f - K_i$; $K_i = 0$

$$W = K_f = \tfrac{1}{2}m_1 v^2 + \tfrac{1}{2}m_2 v^2 = gL\,(m_2 - \mu m_1)$$

$$(m_1 + m_2)\,v^2 = 2gL\,(m_2 - \mu m_1)$$

$$v^2 = 2gL\left(\frac{m_2 - \mu m_1}{m_1 + m_2}\right)$$

$$\Rightarrow \boxed{v = \left[2gL\left(\frac{m_2 - \mu m_1}{m_2 + m_1}\right)\right]^{1/2}}$$

which has units $[\text{m sec}^{-2}\,\text{m}]^{1/2}$ or m sec^{-1}, correct for a speed

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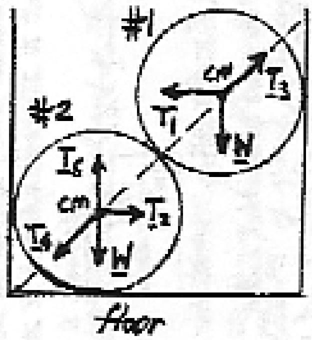
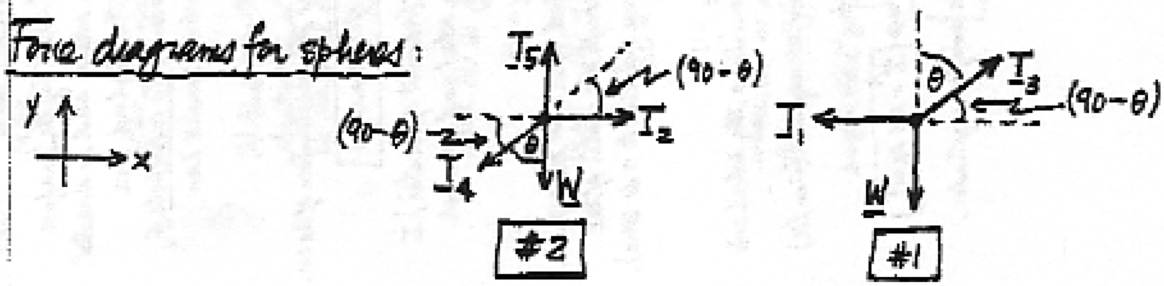
(7)



$\underline{W}$ = weight of each sphere
T_1 = force exerted on #1 by wall (normal to wall)
T_2 = force exerted on #2 by wall (normal to wall)
T_3 = force exerted on #1 by #2 } along line joining their centers of mass
T_4 = force exerted on #2 by #1 }
T_5 = force exerted on #2 by floor (normal to floor)

Force diagrams for spheres:



Conditions for equilibrium: $\sum F_x = 0$; $\sum F_y = 0$

Sphere #1: $\sum F_x = -T_1 + T_3 \cos(90-\theta) = -T_1 + T_3 \sin\theta = 0$ (1)

$\sum F_y = -W + T_3 \cos\theta = 0$ (2)

Sphere #2: $\sum F_x = T_2 - T_4 \cos(90-\theta) = T_2 - T_4 \sin\theta = 0$ (3)

$\sum F_y = -W + T_5 - T_4 \cos\theta = 0$ (4)

Since $\theta = 45^\circ$ and $(90-\theta) = 45^\circ$, equations (1)-(4) become

$T_3 \sin 45^\circ - T_1 = 0 \Rightarrow T_3\left(\frac{\sqrt{2}}{2}\right) - T_1 = 0$ (5)

$T_3 \cos 45^\circ - W = 0 \Rightarrow T_3(\sqrt{2}/2) - W = 0$ (6)

$T_2 - T_4 \sin 45^\circ = 0 \Rightarrow T_2 - (\sqrt{2}/2)T_4 = 0$ (7)

$T_5 - W - T_4 \cos 45^\circ = 0 \Rightarrow T_5 - W - (\sqrt{2}/2)T_4 = 0$ (8)

Also N's 3rd Law: $T_3 = T_4$ (9)

Solving five eqns (5)-(9) for the five unknowns T_1, T_2, T_3, T_4, T_5:

$T_1 = W$

$T_3 = W\sqrt{2}$

$T_4 = W\sqrt{2}$

$T_2 = T_4\frac{\sqrt{2}}{2} = W\sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = W$

$T_5 = W + T_4\frac{\sqrt{2}}{2} = W + W = 2W$

$\boxed{\text{Ans: } T_1 = W;\ T_2 = W;\ T_3 = 1.414W;\ T_4 = 1.414W;\ T_5 = 2W}$