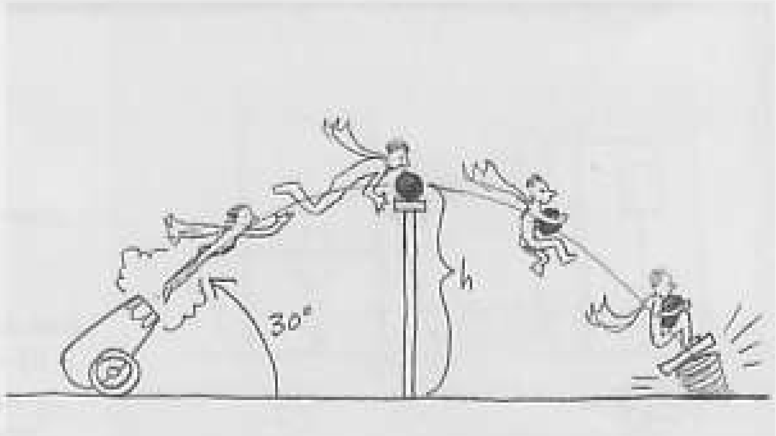
1) Human cannonball (25 points)



The great circus performer, Stupendo Foolini, is shot from a cannon at an angle of 30° from the horizontal, with an initial speed of $v_i = 20$ m/s. At the highest point of his trajectory, Stupendo will grab a ball of mass $M_b = 60$ Kg from its perch atop a tall pedestal. Still holding the ball when he reaches the ground at the end of his flight, Stupendo lands on a platform attached to an angled spring that perfectly cushions his fall. Stupendo has mass $M_s = 60$ Kg, and as always you may use cos(30°) = 0.9 and $g = 10$ m/s^2.

a) What is Stupendo's kinetic energy right as he leaves the cannon?
b) How high, h, does Stupendo get above the ground at his highest point?
c) How fast is Stupendo moving *immediately before* he grabs the ball?
d) How fast is he moving immediately *after* he grabs the ball? Please show your work.
e) How much energy is stored in the spring at the moment he comes to a stop?

a) $K.E._i = \frac{1}{2} m_s v_i^2 = \frac{1}{2} 60\,kg\left(20\,\frac{m}{s}\right)^2 = 30 \cdot 400\,kg\,\frac{m^2}{s^2} = \boxed{12{,}000\,j}$

b) $v_{i,y} = v_i \sin(30°) = v_i \cdot \frac{1}{2} = \frac{20}{2}\frac{m}{s} = 10\,m/s$

const. $\vec{a}$

$a_y = -g$: $0 = v_{f,y} = v_{i,y} + a_y t = 10\,m/s - 10\,m/s^2\, t_f$ so $t_f = \frac{10\,\frac{m}{s}}{10\,m/s^2} = 1\,s$

$y_f = y_i + v_0 t + \frac{1}{2} a t^2$

$= 0 + 10\frac{m}{s} \cdot 1s + \frac{1}{2}(-10\,m/s^2)(1s)^2$

$= 10m - 5m = \boxed{5m}$

c) at highest point, $a_x = 0$ ⇒ $v_{s,x} = v_{s,x_i} = v_i \cos(30°) = 0.9 \cdot 20\frac{m}{s} = 18\frac{m}{s}$

at highest point ⇒ $\vec{v}_s = v_{s,x}\hat{i}$ so $|\vec{v}_s| = \boxed{18\,m/s}$

d) inelastic collision between stupendo & Ball
momentum is conserved:

$\vec{P}_{tot,before} = \vec{P}_{tot,after}$

all motion is in x direction: $m_s v_{s,b} + \cancel{m_B v_{B,i}}^{\,0} = m_s v_{s,a} + m_B v_{B,a}$ $v_{B,a} = v_{s,a}$

$v_{s,a}(m_s + m_B) = m_s v_{s,b}$ $v_{s,a} = \frac{m_s}{m_s + m_B} v_{s,b} = \frac{60kg}{120kg} 18\frac{m}{s}$

$= \frac{1}{2} 18\frac{m}{s}$

$= \boxed{9\,m/s}$

e) cons. of E.

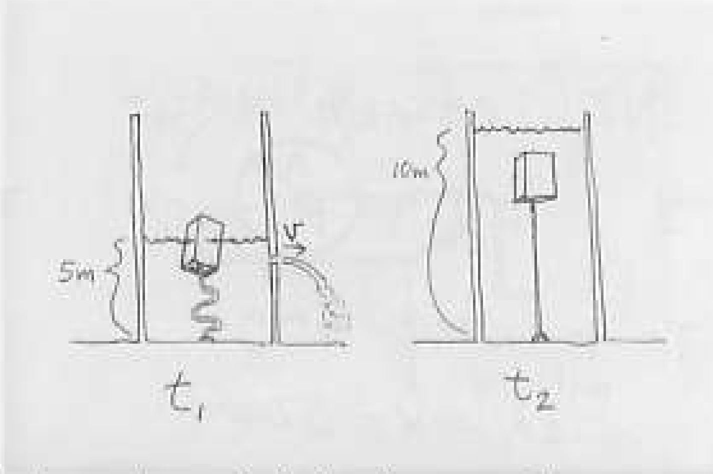
Energy of ball & stupendo at top $= (M_s + M_B)gh + \frac{1}{2}(m_s + m_B)v^2$

E at top $= (120kg)\,10\,m/s^2 \cdot 5m + \frac{1}{2}(120kg)\,9^2\,m^2/s^2 = (120)(50 + \frac{81}{2})j$

$= (120)(90.5)j \approx 10800j \approx \boxed{11{,}000j}$ ← all of this goes to spring P.E.

2) Buoy in a tank (30 points)

At time t_1, 500 Kg buoy floats in a tank of water filled to a height of 5 m. There is a 7 m (massless) rope connecting the buoy to the bottom the tank. Later, water is added to the tank so that at time t_2 the water level has risen to 10 m. The dimensions of the buoy are 1m x 1m x 2m, and the density of water is $\rho = 1000$ kg/m^3.



a) Draw and clearly label a free body diagram of the buoy at time t_1. Include only non-zero forces.
b) What is the buoyant force on the buoy at time t_1?
c) What is the pressure 4.2 m above the bottom of the tank at time t_1? State whether you are using gauge or absolute pressure.
d) If a small hole breaks open 4.2 m above the bottom of the tank, how fast will water rush out?
e) Draw a free body diagram of the buoy at time t_2.
f) What is the tension in the rope at time t_2?

$\uparrow y$

a) $\uparrow F_{Buoyancy}$

$\downarrow F_g = mg$

b) N2L y: $F_{net} = ma$

$F_B - mg = \cancel{ma} \rightarrow 0$ since its not accelerating up or down

$F_B = mg = 500kg \, 10\frac{m}{s^2} = \boxed{5000N}$

c) Bernoulli's eq.: $P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = \text{const.} = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$

$v_1 = v_2 = 0$ $\quad P_{4.2m} + \rho \cdot g \cdot 4.2m = P_{\substack{surface \\ of\ H_2O}} + \rho g \, 5m$

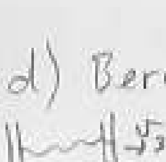
$P_{4.2m} = P_{surface} + 0.8m \, \rho g_{H_2O}$

$= 0\,Pa + 0.8m \cdot 1000\frac{kg}{m^3} \, 10\frac{m}{s^2}$

$\frac{kg\frac{m}{s^2}}{m^2}$

$= (0 + 8000)\frac{kg}{ms^2} = \boxed{8000\,Pa}$ ← gauge Pressure!

d) Bernoulli: $P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2 = P_3 + \rho g h_3 + \frac{1}{2}\rho v_3^2$



$h_2 = h_3$

$8000P_a + \rho g h_2 + \frac{1}{2}\cancel{\rho v_2^2} \rightarrow 0 = 0\,P_a + \rho g h_3 + \frac{1}{2}\rho v_3^2$

out in the air...

$v_3^2 = \frac{2}{\rho_{H_2O}}(8000P_a) = \frac{16000 \, kg/ms^2}{1000 kg/m^3} = 16\frac{m^2}{s^2}$ $\quad v_3 = \sqrt{16\frac{m^2}{s^2}} = \boxed{4\frac{m}{s}}$

e) $\uparrow F_B$

$F_g \downarrow \downarrow T$

f) $F_B = \rho_{H_2O} V_{buoy}\, g = \rho_{H_2O} \, 1m \cdot 1m \cdot 2m \, 10\frac{m}{s^2} = 1000\frac{kg}{m^3} \cdot 2m^3 \cdot 10\frac{m}{s^2} = 20{,}000N$

please see back

$\uparrow y$ N2L: $F_{net_y} = ma_y$

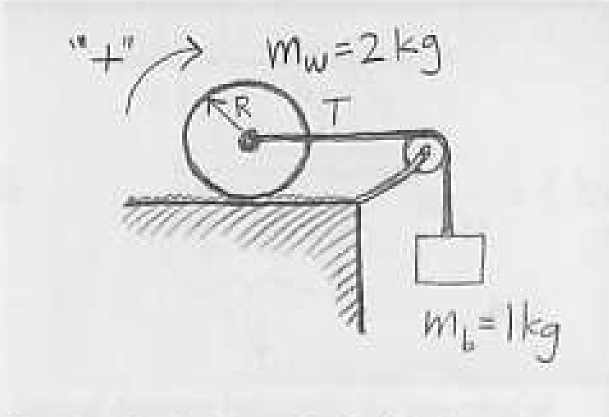
y: $F_B - T - F_g = 0$ $T = F_B - F_g$

$T = +20{,}000N - mg$

$= 20{,}000N - 5000N$

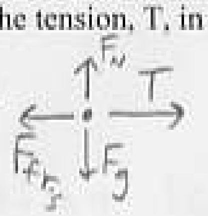
$= \boxed{15{,}000N}$

3) Rolling without slipping (25 points)



A 2 Kg wheel with radius R = 0.5 m is rolling without slipping on a horizontal surface. A 1 Kg hanging block is tied to a massless rope that loops over a massless & frictionless pulley and is tied to the wheel's axle. The moment of inertia of the wheel is $I = ½ M_w R^2$. For this problem, use the convention that clockwise rotations are *positive*.

a) Draw and clearly label two free body diagrams, one for the wheel and one for the block.
b) Write a formula for the torque on the wheel due to the force of friction, F_{fr}. Use the center of the wheel as your pivot point and take care with signs.
c) Find the relationship between the acceleration of the wheel's center of mass and the acceleration of the block. Be specific about the direction of each object's acceleration.
d) What is the tension, T, in the rope?

a) wheel: 

block: 

$\vec{a}_w = (a_{w,x}, a_{w,y})$

F_{fr} is a positive constant

T is a positive constant

b) Friction on wheel: $\boxed{\tau_{fr} = +R\,F_{fr}}$

c) assuming C.O.M. is at center of wheel: $\boxed{a_{wheel,x} = -a_{block,y}}$ ⑥

y ↑ → x

I'll use same axes for wheel & block

d) N2L wheel: $x: \quad T - F_{fr} = a_{w,x}\, m_w$ ①

N2L block: $y: \quad T - m_b g = a_{b,y}\, m_b$ ②

L2L$_\alpha$ wheel: $\tau_{net} = I\alpha$

$$R F_{fr} = \tfrac{1}{2} m_w R^2 \alpha \quad ③$$

$$R\alpha = a_{w,x} \quad ④$$

③&④ ⇒ $F_{fr} = \dfrac{\frac{1}{2} m_w R^2}{R} \dfrac{a_{w,x}}{R} = \frac{1}{2} m_w a_{w,x}$ ⑤

①&⑤ ⇒ $T - \frac{1}{2} m_w a_{w,x} = a_{w,x} m_w \Rightarrow T = \frac{3}{2} m_w a_{w,x}$ ⑧

②&⑥ ⇒ $T - m_b g = -a_{w,x} m_b$

$a_{w,x} = \dfrac{m_b g - T}{m_b}$ ⑦

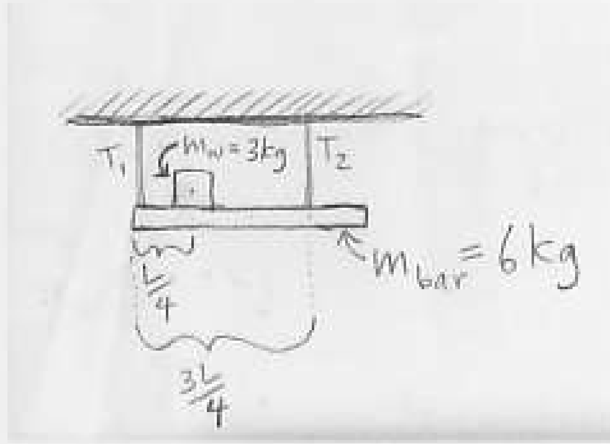
⑦&⑧ ⇒ $T = \frac{3}{2} m_w \dfrac{m_b g - T}{m_b}$

$\dfrac{2}{3}\dfrac{m_b}{m_w} T = m_b g - T$

$T = \dfrac{m_b g}{1 + \frac{2}{3}\frac{m_b}{m_w}} = \dfrac{10}{1+\frac{2}{3}\cdot\frac{1}{2}} N = \boxed{7.5N}$

4) Static equilibrium (20 points)

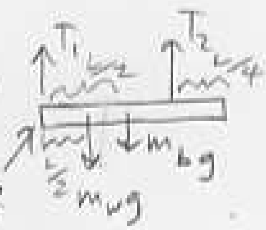
A 6 Kg bar is suspended by two ropes as shown at the right; rope one is at the far left of the beam and rope two is ¾ L from the left side of the beam, where L is the length of the beam. A 3 Kg weight sits ¼ L from the left side of the beam. The beam's mass is evenly distributed.



a) Draw a free body diagram for the beam, and draw a second diagram indicating the direction of each force vector as well as where each force is acting on the beam.
b) What is the tension, T_2, in rope 2?
c) What is the tension, T_1, in rope 1?

a) $T_1 \uparrow\uparrow T_2$ ← F.B.D.
$m_w g \downarrow\downarrow m_b g$

"extended" diagram →



"+"

clockwise positive

b) N2Lα using left of bar as. pivot point

$$\tau_{net} = \cancel{I\alpha}^{\to 0} \quad \alpha = 0 \text{ statics}$$

$$\cancel{T_1}^{\to 0} + g m_w \frac{L}{4} + g m_b \frac{L}{2} - \frac{3L}{4} T_2 = 0$$

(all angles between $\vec{F}$ and $\vec{r}$ are 90°)

$$\frac{3}{2} T_2 = \left(m_w \frac{1}{2} + m_b\right) g$$

$$T_2 = \left(\frac{1}{3} m_w + \frac{2}{3} m_b\right) g = (1\,kg + 4\,kg)\, 10\frac{m}{s^2}$$

$$= 50\,kg\frac{m}{s^2} = \boxed{50N}$$

c) N2L: $T_1 + T_2 - m_w g - m_b g = \cancel{m a_y}^{\to 0}$

$$T_1 = -T_2 + (m_w + m_b) g$$

$$= -50N + (9\,kg)\, 10\frac{m}{s^2}$$

$$= \boxed{40\,N}$$