Physics 8A (sec 1) Sample Exam

The following is the first midterm from my Spring 2000 Physics 8A course. This 80 minute exam covered material from Chapter 8 of RHW which will not be covered on our exam. In particular Problem 4 can be done using conservation of energy or using kinematics. The other questions use only material from RHW chapters 1–7. Solutions are included. The class average was 78%.

RD
2/21/02

PHYSICS 8A - Dr. Dalven
Lec 2(TuTh)
Midterm 1
Thur., Mar 2, 2000, 11:10-12:30 pm

***NOTE: **YOU MUST BE REGISTERED IN LEC 2 TO TAKE THIS EXAM.**

NAME : ___

SID # :___

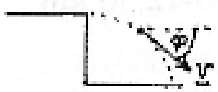
Disc #:___

You may use one sheet no larger than 8.5" by 11" as a memory aid. As always, you must show your work.
CORRECT ANSWERS WITH NO SUPPORTING WORK OR EXPLANATION RECEIVE NO CREDIT!!

1	
2	
3	
4	
5	
6	
TOTAL	

8A MT 1 SP 2000 (1)

(10)(1) A ball is thrown horizontally off a cliff with initial speed v_0. At time t, the velocity vector $\underline{v}$ of the ball makes an angle φ with the horizontal (as shown). Derive an equation giving the function $\varphi(t)$, the angle φ as a function of time t.



8A MT 1 SP 2000 (2)

(20)(2) The coefficient of kinetic friction between a moving body of mass m and a horizontal surface is μ_K. The body is projected along the surface with initial speed v_0 and it eventually comes to rest. (a) If $t=0$ is the time at which the speed of the body is v_0, calculate $v(t)$ giving the speed of the body as a function of time t; (b) Calculate the time t_s it takes for the body to come to rest; (c) Sketch the graph of $v(t)$ as a function of time; (d) Show explicitly that your units in (b) are correct [Part (a) = 8, (b) = (c) = (d) = 4 points].

a)

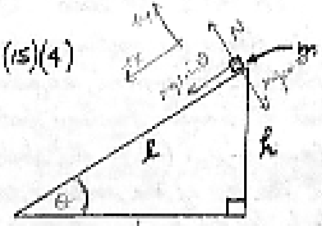
b)

8A MT 1 SP 2000 (3)

(15)(3) A police car, traveling at a constant speed of 95 km/hour is passed by a speeding car traveling at 140 km/hour (in the same direction). Exactly 1.00 second after the speeder passes, the police car accelerates at 2.00 m sec^{-2}. Calculate the time (in sec) required for the police car to overtake the speeding car. Assume that the speed of the speeding car is constant.

8A MT 1 SP 2000 (4)

(15)(4)

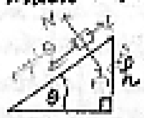


$\sin\theta = \frac{h}{l}$

A frictionless inclined plane is of height h and length l as shown. Starting from rest, a small mass m slides down the plane under the influence of gravity. Calculate the speed v of the mass when it has slid a distance ($l/3$) down the plane.

8A MT 1 SP 2000 (5)

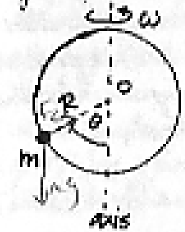
(20)(5) A block of mass m slides down an incline of height h which makes an angle θ with the horizontal, as shown.



The coefficient of kinetic friction between block and plane is μ_K. (a) If the block starts from rest at the top of the incline, calculate the work W done on the block by the frictional force when the block has reached the bottom of the incline. Your answer will be in terms of μ_K, m, g, h, and θ; (b) Explain why the sign of W found in (a) is correct; (c) Calculate the speed v_f of the block at the bottom of the incline. [a=c=8, b=4]

8A MT 1 SP 2000 (6)

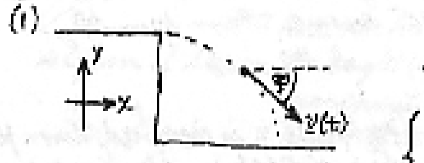
(20)(6) A small bead (with a hole in it) of mass m is free to move without friction on a vertical circular wire loop of radius R. The circular wire loop rotates about a vertical axis through its center O with constant angular speed ω. Calculate the angle θ at which the bead will be in equilibrium (that is, the bead will move neither up nor down.)



(1)

Physics 8A (Sec. 2) Solutions to Midterm #1 March 2, 2000

(1)



Initial velocity $\underline{v}_0$ is horizontal so $\theta = 0$ in

$$\begin{cases} x(t) = x_0 + (v_0 \cos\theta)t \\ y(t) = y_0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2 \end{cases}$$

giving the coordinates $[x(t), y(t)]$ of ball at time t. Then

$$x(t) = x_0 + v_0 t \quad ; \quad y(t) = y_0 - \tfrac{1}{2}gt^2$$

With vector $\underline{v}(t) = [v_x(t)]\hat{i} + [v_y(t)]\hat{j}$, we want to

find: $\begin{cases} v_x(t) = (dx/dt) = v_0 \\ v_y(t) = (dy/dt) = -gt \end{cases}$ since $\begin{cases} x_0 = \text{constant} \\ y_0 = \text{constant} \end{cases}$

Thus $\underline{v}(t) = v_0\hat{i} + (gt)(-\hat{j})$

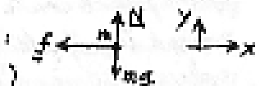
and $\tan\varphi = (v_y/v_x) = (-gt/v_0)$

$\boxed{\varphi = \tan^{-1}(-gt/v_0)}$ gives $\varphi(t)$

Note that, at $t=0$, $\varphi(0) = 0$, agreeing with $\underline{v}_0$ horizontal. As t increases, the magnitude (gt/v_0) of $\tan\varphi$ increases so φ increases as t increases. As drawn above, $y < 0$ so $v_y < 0$ and, since $v_x > 0$, $\tan\varphi > 0$.

8A MT 1 SP 2000 SOLNS (2)

(2) (a) Force diagram of mass m :



$\underline{N}$ = normal force exerted on m
$m\underline{g}$ = weight of m
$\underline{f}$ = frictional force exerted on m

Since vertical velocity of mass m has constant value zero,

$\Sigma F_y = ma_y = 0 = N - mg \Rightarrow \boxed{N = mg}$

The only horizontal force acting on m is $f = \mu_k N = \mu_k mg$ so Newton's 2nd law is (with $\underline{f}$ in (−x) direction):

$$-\mu_k mg = m\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = \frac{dv}{dt} = -\mu_k g$$

Then $dv = -\mu_k g\, dt \Rightarrow v(t) = -\mu_k g t + C$

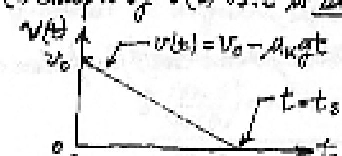
To find C : When $t = 0$, $C = v(0) \equiv v_0$ = initial velocity, so

$\boxed{v(t) = v_0 - \mu_k g t}$ $\mu_k > 0$

meaning velocity v decreases with increasing time t.

(b) Body stops when $v = 0 \Rightarrow 0 = v_0 - \mu_k g t_s \Rightarrow \boxed{t_s = (v_0/\mu_k g)}$

(c) Graph of $v(t)$ vs. t is straight line : $v = 0$ when $t = t_s$



(d) In (b), v_0 = m sec^{-1}, μ_k = dimensionless g = m sec^{-2} so $(v_0/\mu_k g)$ is in seconds, correct unit for time t_s

(3) For police car to overtake speeding car, the distance d_s moved by speeding car in time t (after speeder passes police car) must equal distance d_p moved by police car in time t:

$$d_s = d_p$$

$$v_s t = v_0 t + \tfrac{1}{2} a_p (t-1)^2$$

where v_0 is initial velocity of police car, v_s is constant velocity of speeding car, a_p is (constant) acceleration of police car and $(t-1)$ is time interval during which police car accelerates. We have $v_0 = 95$ km/hr $= 26.39$ m sec^{-1}, $v_s = 140$ km/hr $= 38.89$ m sec^{-1}, and $a_p = 2$ m sec^{-2}. Then

$$(38.89)t = (26.39)t + (t^2 - 2t + 1)$$

$$t^2 - 12.5t - 2t + 1 = 0$$

$$t^2 - 14.5t + 1 = 0$$

Solve for t:

$$t = \frac{14.5 \pm [210-4]^{1/2}}{2} = \frac{14.5 \pm 14.36}{2}$$

$$t = 14.43 \text{ sec and } t = 0.07 \text{ seconds}$$

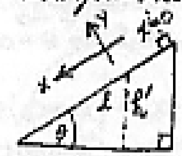
We want $(t-1)$ to be positive, so we choose

$$\boxed{t = 14.43 \text{ sec}}$$

as time (after speeder passes police car) for police car to overtake speeder.

(4) There are two ways to do this problem, either using energy conservation or using kinematics ($F=ma$). Either way will give the correct answer. From geometry

$$h = \ell \sin\theta$$



Coordinate x is measured down plane. When $x = (\ell/3)$, $h' = (2\ell/3)\sin\theta$

Since plane is frictionless, total mechanical energy $E = K + U$ (U = gravitational potential energy) is conserved so

$$E = K(\text{top}) + U(\text{top}) = K(x=\tfrac{\ell}{3}) + U(x=\tfrac{\ell}{3})$$

$$0 + mgh = \tfrac{1}{2}mv^2 + mgh'$$

$$mgh - (2mg\ell/3)\sin\theta = \tfrac{1}{2}mv^2$$

But $\sin\theta = (h/\ell)$, so $mgh - (2/3)mgh = \tfrac{1}{2}mv^2$

$$\Rightarrow \tfrac{1}{3}gh = \tfrac{1}{2}v^2 \Rightarrow \left(\frac{2gh}{3}\right) = v^2 \Rightarrow \boxed{v = (2gh/3)^{1/2}}$$

when $x = (\ell/3)$

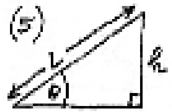
To use kinematics, the net acceleration in x direction (down the plane) is $(g\sin\theta)$ since only gravity acts down plane. Then, when $(x - x_0) = (\ell/3)$,

$$v^2 = v_0^2 + 2a(x-x_0) \Rightarrow v^2 = 0 + (2g\sin\theta)(\ell/3)$$

$$v^2 = (2g\ell/3)\sin\theta = (2g\ell/3)(h/\ell)$$

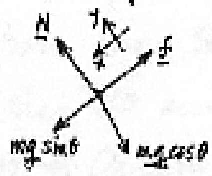
$$v^2 = (2gh/3) \Longrightarrow \boxed{v = (2gh/3)^{1/2}}$$

8A MT 1 SOLNS SP 2000 (5)

(5) 

Let L = length of incline $= \frac{h}{\sin\theta}$

(a) To find the frictional force $\underline{f}$, draw force diagram of block; the weight ($m\underline{g}$) has been resolved into a component ($mg\sin\theta$) parallel to the incline and ($mg\cos\theta$) normal to the incline.



Since $(\underline{N} + m\underline{g}\cos\theta) = 0$, $N = mg\cos\theta$ and

$f = |\underline{f}| = \mu_k N = \mu_k mg\cos\theta$

The frictional force vector $\underline{f} = f(-\hat{\imath}) = (-\mu_k mg\cos\theta)\hat{\imath}$ and the displacement vector of the block is $\underline{L} = L\,\hat{\imath}$ (since $\underline{f}$ and $\underline{L}$ are antiparallel. The work W done by force $\underline{f}$ is, since $(\hat{\imath}\cdot\hat{\imath}) = 1$,

$$W = \underline{f}\cdot\underline{L} = (-\mu_k mg\cos\theta)(L)(\hat{\imath}\cdot\hat{\imath}) = -\mu_k mgL\cos\theta$$

But, from figure, $L = h/(\sin\theta)$, so $\boxed{W = -\mu_k mgh\cot\theta}$ since $[(\cos\theta)/(\sin\theta)] = \cot\theta$.

(b) From $W = -\mu_k mgh\cot\theta$, $W < 0$ ($\mu_k, m, g, h, \cot\theta$ are all positive). W is negative because the direction of $\underline{f}$ is opposite to $\underline{L}$.

(c) Since friction is a non-conservative force, total energy $E = (K+V)$ where V is gravitational potential energy of block, is not conserved. However, taking $V(\text{bottom}) \equiv 0$, we have

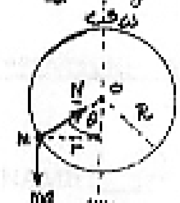
$$E(\text{bottom}) = E(\text{top}) + W \implies \tfrac{1}{2}mv_b^2 + 0 = 0 + mgh + W$$

so $\frac{1}{2}mv_b^2 = mgh - \mu_k mgh\cot\theta \implies v_b^2 = 2gh(1 - \mu_k\cot\theta)$

$\boxed{v_b = [2gh(1-\mu_k\cot\theta)]^{1/2}}$ is speed of block at bottom of incline

8A MT 1 SOLNS SP 2000 (6)

(6) As the loop spins, there are two forces exerted on the bead:
$\underline{N}$ = radially directed normal force exerted on bead by loop
$m\underline{g}$ = weight (gravitational force) exerted on bead. Taking



the vertical axis as the y-axis, ($m\underline{g}$) is in the $(-y)$ direction and the horizontal and vertical components of $\underline{N}$ are

$N_x = N\cos(90-\theta) = N\sin\theta$ is horizontal comp.
$N_y = N\cos\theta$ is vertical component

The motion of the bead is in a horizontal circle (normal to the plane of the paper) of radius r where

$$r = R\cos(90-\theta) = R\sin\theta$$

where R is the radius of the spinning loop. Since the bead moves in a circular path, which in a top view, is shown at left, and where the centripetal force exerted on the bead is N_x. Further, since the bead does not move vertically, we have for equilibrium,

(TOP VIEW)

$N_y - mg = 0 \implies N\cos\theta = mg \implies N = \frac{mg}{(\cos\theta)}$

so $N_x = N\sin\theta = \frac{mg(\sin\theta)}{(\cos\theta)} = mg\tan\theta$

Then the centripetal force exerted on mass m moving in circle of radius r at angular speed ω is $(mr\omega^2)$, with $r = R\sin\theta$, so

$$N_x = mg\tan\theta = m\omega^2 R\sin\theta$$

$$g\left(\frac{\sin\theta}{\cos\theta}\right) = \omega^2 R\sin\theta \implies \frac{g}{\cos\theta} = \omega^2 R$$

$\boxed{\cos\theta = (g/\omega^2 R)}$ is the value of angle θ for which bead is in equilibrium.