University of California at Berkeley
Department of Physics
Physics 8A, Fall 2009

Midterm 1
Oct 8, 2009

You will be given 100 minutes to work this exam. No books are allowed, but you may use a handwritten formulae sheet no larger than one side of an 8 ½" by 11" sheet of paper. No electronics of any type (cell phones, calculators, etc) are allowed. Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Please use the symbols described in the problems, tell us why you're writing any new equations, and label any drawings that you make. Write your answers directly on the exam, and if you have to use the back of a page or the blank sheet from the back of the exam make sure to put a note on the front. Do not use a blue book or any extra scratch paper.

$$\vec{v} = d\vec{x}/dt \quad \vec{a} = d\vec{v}/dt \quad x(t) = x_0 + v_0 t + \frac{1}{2}a_0 t^2 \quad v(t) = v_0 + a_0 t \quad v^2(x) = v_0^2 + 2ax$$

$$\sum \vec{F} = m\vec{a} \quad F = mv^2/R \quad F = mg \quad \vec{P} = \sum m_i \vec{v}_i \qquad g = 10 \text{ m/s}^2$$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \sum m_i x_i \Big/ \sum m_i \quad v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \sum m_i v_i \Big/ \sum m_i$$

$$F_{fr} = \mu_k F_N \quad F_{fr} \le \mu_s F_N \quad W = \vec{F} \bullet \vec{x} \quad \vec{P} = m\vec{v} \quad F_{W.R.} = \frac{1}{2} C \rho A v^2 \quad b = l\cos\theta \quad a^2 + b^2 = c^2$$

$\sin 45° = \cos 45° = 0.7$ $\qquad \cos 60° = \sin 30° = 0.5$ $\qquad \sin 60° = \cos 30° = 0.9$

NAME: Mike DeWeese

SID NUMBER: ______________________

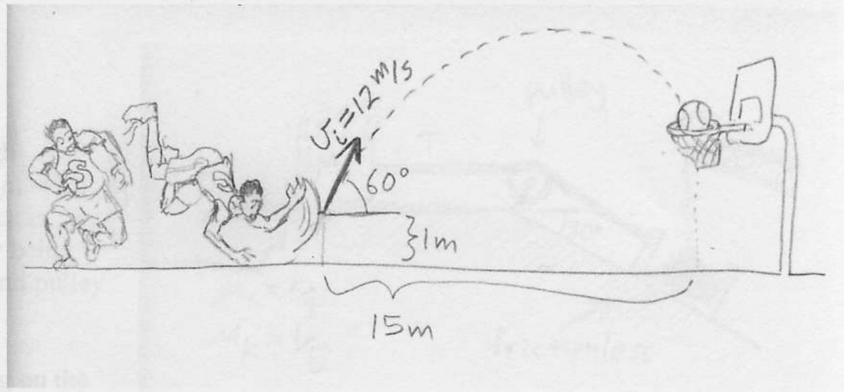
DISCUSSION SECTION NUMBER: ______________

DISCUSSION SECTION DATE/TIME: ____________

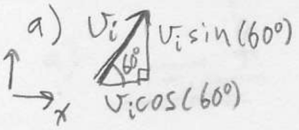
NAME OF YOUR DISCUSSION GSI: ____________

1	
2	
3	
4	
Total	

1) (25 points) Fast Break.
Stanford has the basketball and they're up by a point, but in the final second of the game a Cal player dives for the ball and manages to hurl it upwards from a height of only 1 *m* above the ground at a steep angle of 60° from the horizontal with an initial speed of $v_i = 12$ *m/s*. The basketball is 0.5 *kg*, and the basket is located 15 *m* away from the Cal player as shown in the diagram. You may use sin(60°) = cos(30°) = 0.9.



a) What is the magnitude of the horizontal component of the ball's initial velocity?
b) How much time does it take the ball to reach the basket?
c) What is the kinetic energy of the ball at its highest point?
d) How high is the ball above the ground at its highest point?
e) What is the direction and magnitude of the ball's acceleration right before it lands in the basket?

a) 

horizontal component of velocity: $v_{ix} = v_i \cos(60°) = \frac{1}{2} \cdot 12 \, m/s = \boxed{6 \, m/s}$

b) $a_x = 0 \Rightarrow$ const. v_x so $\Delta x = v_x \Delta t \Rightarrow$ time to reach basket $= \Delta t = \frac{\Delta x}{v_x} = \frac{15m}{6 \, m/s} = \boxed{2.5s}$

c) at highest point, $v_y = 0$ so $|\vec{v}| = v_x = v_{x_0}$ since $a_x = 0$

$K.E._{top} = \frac{1}{2} m v_{x_0}^2 = \frac{1}{2} \frac{1}{2} kg (6 \, m/s)^2 = 3 \cdot 3 j = \boxed{9j}$

d) cons. of E: $(KE + PE_g)_{top} - (KE + PE_g)_i = \cancel{W_{on}} \to 0$ ← since I am accounting for gravity w/ PE_g

$9j + mg y_{top} - (\frac{1}{2} m v_i^2 + mgy_i) = 0$

$mg y_{top} = -9j + \frac{1}{2} m v_i^2 + mgy_i$

$y_{top} = -\frac{9j}{mg} + \frac{1}{2} \frac{\cancel{m} v_i^2}{\cancel{m} g} + \frac{\cancel{mg}}{\cancel{mg}} y_i$

$y_{top} = -\frac{9j}{\frac{1}{2} kg \, 10 \, m/s^2} + \frac{(12 \, m/s)^2}{2 \, 10 \, m/s^2} + 1m$

$= -\frac{9}{5} m + \frac{144}{2 \cdot 10} m + 1m$

$= -1.8m + \frac{72}{10} m + 1m$

$= (-1.8 + 7.2 + 1.0) m$

$= \boxed{6.4 \, m}$

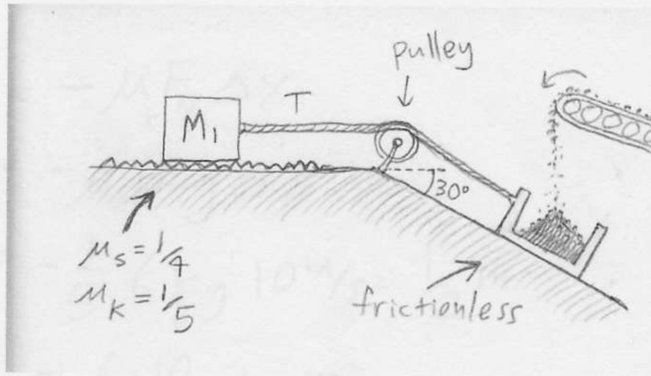
e) F.B.D. of ball $\downarrow mg$ $\quad F_{net_y} = ma_y \quad -\cancel{m}g = \cancel{m}a_y$

$a_y = -g$ so $|\vec{a}| = \boxed{g = 10 \, m/s^2}$

straight down

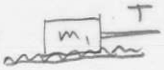
2) (25 points) Sandbox on an incline.

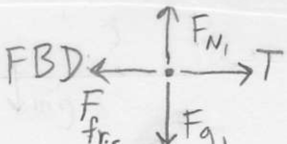
A block with mass $M_1 = 6\ kg$ is initially at rest on a rough horizontal surface with a static coefficient of friction of $\mu_s = 1/4$ and a kinetic coefficient of friction of $\mu_k = 1/5$. The block is connected with a rope to an open box that is very slowly being filled with sand on a frictionless 30° incline. The rope and pulley are ideal.



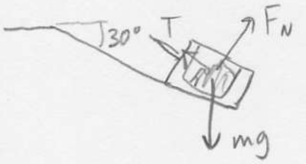
a) Draw a free body diagram to indicate *all* forces acting on the *block* M_1 before it starts to move.
b) Draw a free body diagram for the box of sand.
c) What is the total mass M_2 of the box of sand right when the box starts to slip down the surface?
d) How much work does friction do on the block as the block slides 0.5 *m*?
e) How much work does gravity do on the box of sand as the box slides 2.0 m along the slope?

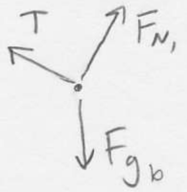
a)





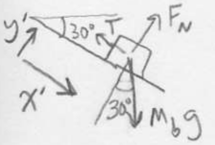
b)



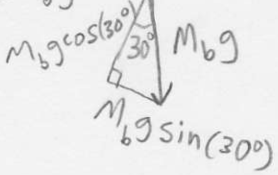


c)

N2L for M_1 x: $-F_{fr_s} + T = \cancel{M_1 a_{x_1}}^{0} \Rightarrow F_{fr_s} = T$ — ①



N2L for Box x': $-T + \cancel{F_{N_{x'}}}^{0} + M_b g \sin(30°) = \cancel{M_b a_{x'}}^{0}$



$T = M_b g \sin(30°) = M_b\ 10\,m/s^2\ \frac{1}{2} = M_b \cdot 5\,\frac{m}{s^2}$ ②

① & ② $\Rightarrow F_{fr_s} = M_b \cdot 5\,m/s^2$ — ③

$F_{fr_s} \le \mu_s F_{N_1}$

$\le \frac{1}{4} M_1 g$

$\le \frac{1}{4}\ 6\,kg\ 10\,m/s^2 = 15\,N$

so the maximum force of friction is 15N

plug that into eq. ③:

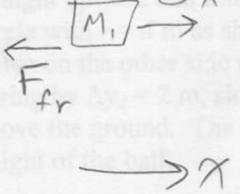
$F_{fr_s\,max} = 15N = M_b \cdot 5\,m/s^2$

so $M_b = \frac{15N}{5\,m/s^2} = \boxed{3\,kg}$

See back for d) & e)

Problem 2 continued:

d) $W_{fr\,on\,block} = \vec{F}_{fr} \cdot \Delta\vec{X}_{block} = F_{fr_x} \Delta X = -\mu_k F_N \Delta X$

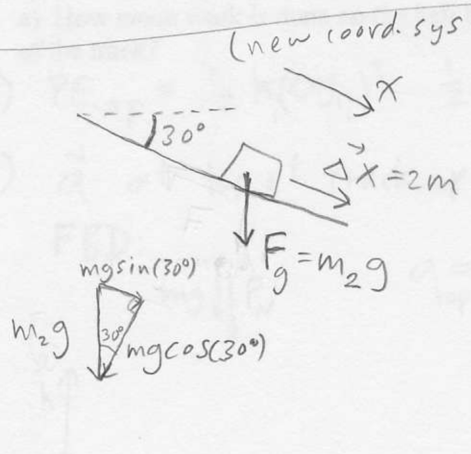


$$= -\mu_k m_1 g \Delta X$$

$$= -\frac{1}{5} 6\,kg\, 10\,\tfrac{m}{s^2}\, \tfrac{1}{2} m$$

$$= -\frac{6 \cdot 10}{2 \cdot 5}\, kg \frac{m^2}{s^2}$$

$$= \boxed{-6\,J}$$ ← work is negative because Force of friction opposes the motion

e) 

$W_{g\,on\,sandbox} = \vec{F}_{g\,on\,box} \cdot \Delta\vec{X}_{box}$

$$= m_2 g \sin(30°)\, \Delta X_{box}$$

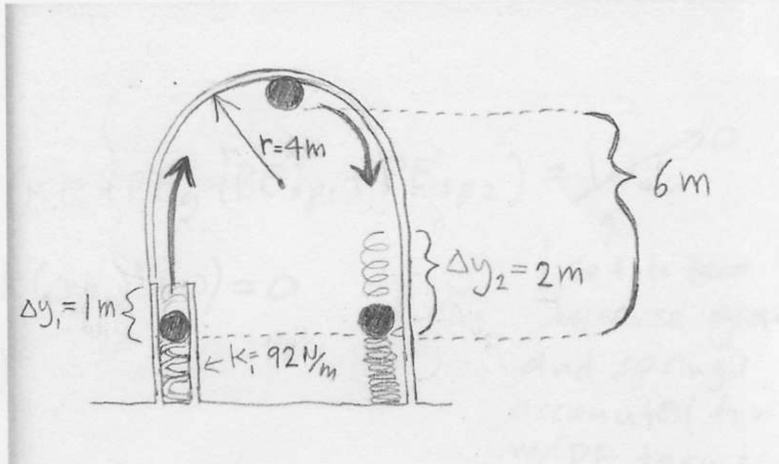
$$= 3\,kg\, 10\,\tfrac{m}{s^2}\, \tfrac{1}{2}\, 2m$$

$$= 30\, kg\, \tfrac{m^2}{s^2}$$

$$= \boxed{30\,J}$$ ← positive because force of gravity has a positive component in the direction of motion down the slope

I use the mass I found in part C because the sand is only being added to the box <u>very</u> slowly so the mass won't change hardly at all while the box slides down the slope

3) (25 points) Spring gun and frictionless track.
A spring with spring constant $k_1 = 92$ N/m is compressed by $\Delta y_1 = 1$ m and then released, which shoots a 1/2 kg ball straight upward and into a frictionless track that forms a half-circle with $r = 4$ m as shown in the diagram. The ball comes down on the other side of the ramp and compresses a second spring by $\Delta y_2 = 2$ m, slowing it to a stop at its initial height above the ground. The top of the track is 6 m above the initial height of the ball.



a) How much potential energy is stored in spring 1 before the ball is shot upwards?
b) When the ball is at the top of the curved track, what is the *direction* and *magnitude* of its acceleration?
c) What is the *direction* and *magnitude* of the normal force acting on the ball at the top of the track?
d) What is the spring constant k_2 of the second spring?
e) How much work is done on the ball by the track from the time it is shot upwards by the spring until it reaches the top of the track?

a) $PE_{sp_{1i}} = \frac{1}{2} k_1 (\Delta y_{1,i})^2 = \frac{1}{2}\, 92 \frac{N}{m} (1m)^2 = \boxed{46\ Nm} = 46\ J$

b) $\vec{a}$ at top of track points $\boxed{\text{straight down}}$ since all forces point down.

FBD: $mg \downarrow \downarrow F_N$

$a_{top} = \frac{v_{top}^2}{r}$

find v_{top}: cons. of Energy:

$(KE + \cancel{PE_{sp}}^{0} + PE_g)_f - (\cancel{KE}^{0} + PE_{sp} + PE_g)_i = \cancel{W_{on}}^{0}$

$\frac{1}{2} m v_{top}^2 + m g y_{top} - \frac{1}{2} k (\Delta y_{1,i})^2 + m g y_i = 0$

$\frac{1}{2} m v_{top}^2 = m g (y_i - y_{top}) + \frac{1}{2} k (\Delta y_{1,i})^2$

$v_{top} = \sqrt{\frac{\cancel{m} g}{\cancel{m}} 2 (y_i - y_{top}) + \frac{\cancel{2}}{\cancel{2}} \frac{k}{m} (\Delta y_{i,i})^2}$

$= \sqrt{20 m/s^2 (-6m) + \frac{92\ N/m}{1/2\ kg} (1m)^2}$

$= \sqrt{-120\ m^2/s^2 + 184 \frac{\cancel{kg}\,\cancel{m}/s^2}{\cancel{kg}\,\cancel{m}} m^2}$

$= \sqrt{64\ m^2/s^2}$

$= 8\ m/s$

SO: $a_{top} = \frac{v_{top}^2}{r} = \frac{(8 m/s)^2}{4m} = \frac{64\ m^2/s^2}{4m} = \boxed{16\ m/s^2}$ downward

c) N2L: y: $+F_g + F_N = +m a_y$

$mg + F_N = m \frac{v^2}{r}$

$F_N = m\left(\frac{v^2}{r} - g\right) = \frac{1}{2} kg (16 m/s^2 - 10 m/s^2)$

$= \frac{1}{2} (6)\, kg\, m/s^2 = \boxed{3\ N}$ downward

See back

Problem 3 cont.:

d) cons. of E:

$$(KE + PE_g + PE_{sp,1} + PE_{sp,2})_f - (KE + PE_g + PE_{sp,1} + PE_{sp,2})_i = \cancel{W_{on}} \to 0$$

set to zero because grav. and springs accounted for w/ P.E. terms AND track can't do work on ball because track is frictionless so $F_{track} = F_N$ only

$$0 + 0 + 0 + \tfrac{1}{2}k_2(\Delta y_2)^2 - (0 + 0 + \tfrac{1}{2}k_1(\Delta y_1)^2 + 0) = 0$$

$$\cancel{\tfrac{1}{2}}k_2(\Delta y_2)^2 = \cancel{\tfrac{1}{2}}k_1(\Delta y_1)^2$$

$$k_2 = k_1 \frac{(\Delta y_1)^2}{(\Delta y_2)^2}$$

$$= 92\,\tfrac{N}{m}\,\frac{(1m)^2}{(2m)^2}$$

$$= 92\,\tfrac{N}{m}\left(\tfrac{1}{4}\right)$$

$$= \boxed{23\ N/m}$$

$92 \div 4 = 23$

e) $W_{on} = \vec{F}_{on} \cdot \Delta\vec{x}$ but $\vec{F}_{track\ on\ ball}$ is a normal force since the track is frictionless, so

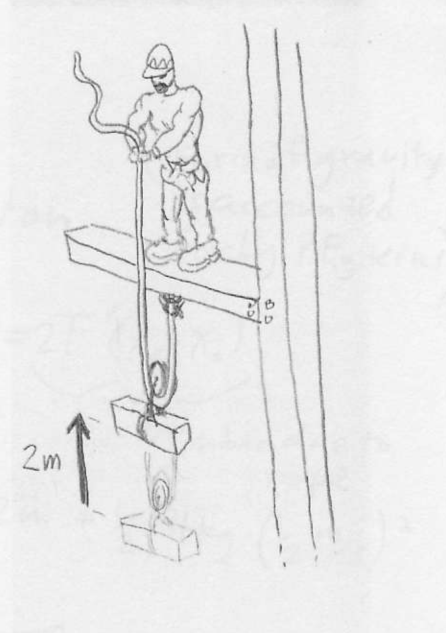
$\vec{F}_{track\ on\ ball} = \vec{F}_N$ and $\vec{F}_N \cdot \Delta\vec{x} = 0$ everywhere along the balls trajectory so:

$$\boxed{W_{track\ on\ ball} = 0\,J}$$

track does no work!

4) (25 points) Raising the bar

A construction worker is raising a 4 kg bar by pulling upward on a massless rope that passes around an ideal pulley mounted to the bar; the rope is tied to the beam he's standing on. The bar rises at a constant speed of 0.5 m/s as it moves a distance of $\Delta x = 2$ m.



a) Draw and clearly label a free body diagram of the bar.
b) What is the tension in the rope?
c) How much work was done by the construction worker on the rope?
d) Now, without stopping the bar, the worker pulls with twice as much force as he was pulling before, and the bar rises another 2 m. What is the direction and magnitude of the bar's acceleration?
e) What is the final kinetic energy of the bar?

a) FBD for bar: $T \uparrow \uparrow T$ $\quad \uparrow x$

$\downarrow F_g = mg$

b) N2L: x: $T + T - mg = \underbrace{\cancel{m a_x}}_{\to 0}$ since velocity is constant

$$2T = mg$$
$$T = \tfrac{1}{2} mg$$
$$T = \tfrac{1}{2}\, 4\,\text{kg}\; 10\,\text{m/s}^2$$
$$T = \boxed{20\text{ N}}$$

c) $W_{\text{man on rope}} = F_x \cdot \Delta X_{\text{rope}}$

due to the pulley arrangement, twice as much rope must be lifted as the distance the bar moves. so $\Delta X_{\text{rope}} = 2\Delta X_{\text{bar}}$

$$= F_{\substack{\text{man}\\\text{on}\\\text{rope}}} \cdot 2\Delta X_{\text{bar}}$$
$$= T\, 2\, \Delta X_{\text{bar}}$$
$$= (20\text{N})(2)(2\text{m})$$
$$= 80\text{ Nm} = \boxed{80\text{ j}}$$

d) FBD of bar $T' \uparrow \uparrow T'$ $\quad T' = 2T$ because man is now pulling 2 times as hard

$\downarrow mg$

so N2L: $F_{\text{net on bar}_x} = m a_{\text{bar}_x}$

$$T' + T' - mg = m a_{\text{bar}x}$$
$$2T' - mg = m a_x$$
$$4T - mg = m a_x \Rightarrow a_x = \frac{4T}{m} - g = \frac{4\frac{mg}{2}}{m} - g = 2g - g = g$$

so $\boxed{a = 10\,\text{m/s}^2 \text{ upwards}}$

e) See back

Problem 4 cont.:

e)

conservation of E:

$$(KE + PE_g)_f - (KE + PE_g)_i = W_{on}$$

(Force of gravity is accounted for by $P.E_g$ terms)

$$KE_f + mgx_f - \left(\tfrac{1}{2}mv_i^2 + mgx_i\right) = \underbrace{2T'(x_f - x_i)}_{W_{\text{on bar due to rope}}}$$

$$K.E_f = 2T'\Delta X - mg\Delta X + \tfrac{1}{2}mv_i^2$$

$$= 2 \cdot 40N \cdot 2m - 4kg \cdot 10\,m/s^2 \cdot 2m + \tfrac{1}{2} \cdot 4kg \cdot \left(\tfrac{1}{2}\,m/s\right)^2$$

$$= 160J - 80J + \tfrac{1}{2}J$$

$$= \left(80 + \tfrac{1}{2}\right)J = \boxed{80.5J} \approx 80J$$

Here's another way to do it:

const. a: $a = +10\,m/s^2$ (from part d) $v_i = \frac{1}{2}\,m/s$ $\Delta X = 2m$

const. a: $v_f^2 = v_i^2 + 2a\Delta X$

$$v_f^2 = \left(\tfrac{1}{2}\,m/s\right)^2 + 2\left(10\,m/s^2\right)(2m)$$

$$v_f^2 = \tfrac{1}{4}\,m^2/s^2 + 40\,m^2/s^2 = 40.25\,m^2/s^2 \approx 40\,m^2/s^2$$

$$K.E_f = \tfrac{1}{2}mv_f^2$$

$$= \tfrac{1}{2} \cdot 4kg \cdot 40.25\,m^2/s^2$$

$$= 80.5\,kg\,m^2/s^2$$

$$\approx \boxed{80J}$$