Physics 8A Section 2 Midterm # 2

Lecture 2

Your name

Solutions

April 10, 2003

Discussion session number / GSI name

1) DON'T OPEN THIS EXAM UNTIL INSTRUCTED TO BEGIN
2) Sit one seat away from anyone else.
3) Do all your work on the page indicated for each problem.
4) Show all work; full credit will require both the correct answer and your reasoning.
5) This is a closed book exam but calculators and one sheet of notes are allowed.
6) Possibly useful equations include:

$F = dp / dt = m a$
$p = mv$
$F_c = m v^2 / r$
$x = x_0 + v_0 t + 1/2\, a t^2$
$W = F x$
$U = m g h$
$U = 1/2\, kx^2$
$K = 1/2\, m v^2$
$\tau = I \alpha$
$I = mr^2$
$\tau = r \times F$

$L = I \omega$
$\tau = dL/dt$
$v = \omega r$
$a = \alpha r$
$I = I_{com} + mh^2$
$K = 1/2\, I \omega^2$
$v_1 A_1 = v_2 A_2$
$p + 1/2\, \rho v^2 + \rho g y = \text{constant}$
$\rho_{H2O} = 1000 \text{ kg} / \text{m}^3$

SCORING - we'll handle this space :)

1)

2)

3)

4)

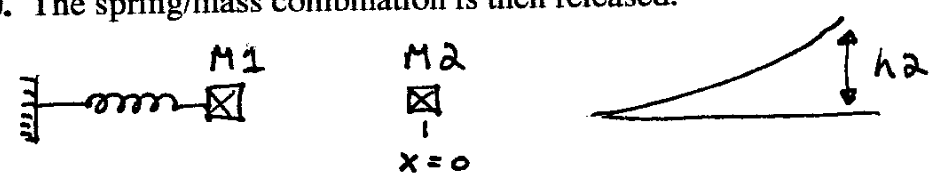
5)

TOTAL 100

1) A massless spring has a block of mass **m1 = 5 kg** attached to it. The mass can oscillate without friction in a horizontal plane. The period **T** of oscillation of the spring/mass combination is **T = π s** (or 3.14 seconds). In a first experiment, the spring/mass combination is compressed by a distance **x = 1 m,** away from its equilibrium point **x = 0**, and then released.

a) Write equations for the resulting position **x (t)** and velocity **v (t)** of mass **m1** as a function of time.

In a second experiment, the spring/mass combination is again compressed by a distance **x = 1 m,** away from its equilibrium point, **x = 0**. A second free mass **m2 = 10 kg** is then placed at **x = 0.** The spring/mass combination is then released.



b) What is the velocity of mass **m1** just before it hits mass **m2**?

c) If mass **m1** sticks to mass **m2**, what is the maximum kinetic energy of the combined oscillating masses?

d) If mass **m1** collides elastically with mass **m2**, what vertical height **h2** is reached by **m2** as it slides up the indicated hill?

(a) $T = 2\pi/\omega$ so $\omega = 2$ radians/sec

$X(t) = +(1\text{ meter}) \cos(2\text{ rad/sec} \cdot t + \pi)$

or $+(1\text{ meter}) \cos(2\text{ rad/sec} \cdot t - \pi)$

or $-(1\text{ meter}) \cos(2\text{ rad/sec}\ t)$

$v(t) = -2\text{ m/s} \cdot \sin(2\text{ rad/sec} \cdot t + \pi)$

or $-2\text{ m/s} \cdot \sin(2\text{ rad/sec} \cdot t - \pi)$

or $2\text{ m/s} \cdot \sin(2\text{ rad/sec})$

- EXTRA WORK PAGE -

b). at $x=0$, $V = V_{max} = \omega X_m = \boxed{2 m/s.}$

c) $P_i = P_f$

$m_1 v_1 + m_2 v_2 = m_1 v_1 + m_2 v_2$

$5kg \cdot 2 m/s + 0 = (5kg + 10kg) V_f$

$\frac{10}{15} m/s. = V_f$

$\frac{2}{3} m/s. = V_f$

$KE = \frac{1}{2} m v^2$

$= \frac{1}{2}(10+5 kg)\left(\frac{10}{15} m/s\right)^2$

$\boxed{KE = 10/3 \text{ Joules } (3.33 \text{ Joules})}$

d) $V_{2f} = \frac{2m_1}{m_1+m_2} V_{1i}$ (elastic collision)

$V_{2f} = \frac{10kg}{15kg} 2m/s = \frac{4}{3} m/s.$

Using conservation of Energy,

$U_i + K_i = U_f + K_f$

$0 + \frac{1}{2} \cancel{m_2} \left(\frac{4}{3} m/s\right)^2 = \cancel{m_2} (10 m/s^2)(h_2) + 0$

$\frac{8}{9} m^2/s^2 = 10 m/s^2 \, h_2$

$h_2 = \frac{8}{90} m$

$\boxed{h_2 = \frac{4}{45} m \ (0.089 m)}$

2) A piece of stuff with mass $\mathbf{m_s = 5\ kg}$ is stuck on the side of stationary wheel whose mass $\mathbf{m_w = 10\ kg}$ is concentrated in its rim. The wheel has a radius $\mathbf{r = 2\ m}$ and it is lying horizontally and has a frictionless bearing. Then, a force **F** is applied tangent to the edge of the wheel for a duration of **2 s** to get it rotating at **2 rotations per second.** Next, the stuff flies off. Finally, a force is applied tangent to the edge of the wheel for a duration of **2 s** to stop the wheel.

$\omega = 2\,rot/sec \cdot \frac{2\pi\,rad}{1\,rot} = -4\pi\,rad/sec$ (clockwise)

a) <u>What is the speed of the stuff as it leaves the wheel?</u>

b) <u>How big was the force that was applied to get the wheel rotating?</u>

c) <u>How big was the force that was applied to stop the wheel.</u>

a) $V = \omega r$

$= \left(\frac{2\,rot}{sec} \cdot \frac{2\pi\,rad}{1\,rot}\right)(2m)$

$\boxed{V = 8\pi\ m/s\ (25.13\ m/s)}$

b) $\tau_{net} = (\vec{F} \cdot d)_{net} = I_{net}\,\alpha$

$d = 2m$, $I_{net} = I_{wheel} + I_{stuff}$

$= m_w(2m)^2 + m_s(2m)^2$

$I_{net} = 60\ kg \cdot m^2$

$\omega = \omega_0 + \alpha t$

$-4\pi\,\frac{rad}{sec} = 0 + \alpha \cdot 2\,sec$

$\alpha = -2\pi\ rad/sec^2$

$(\vec{F} \cdot d)_{net} = I_{net}\,\alpha$

$-(F \cdot 2m) = 60\,kg \cdot m^2 \cdot -2\pi\,rad/sec^2$

$\boxed{F = 60\pi\ N\ (188.5\,N)}$

c) $(\vec{F} \cdot d)_{net} = I_{net}\,\alpha.$

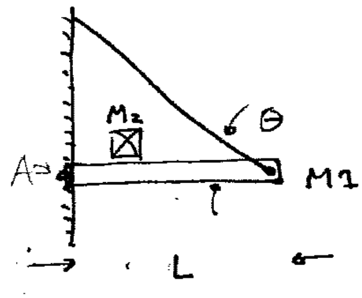
$d = 2m$, $I_{net} = I_{wheel} = 40\,kg \cdot m^2$, $\alpha = +2\pi\ rad/sec^2$

$(\vec{F} \cdot d)_{net} = I_{net}\,\alpha.$

$+(F \cdot 2m) = 40\,kg \cdot m^2 \cdot 2\pi\,rad/sec^2$

$\boxed{F = 40\pi\ N\ (125.7\,N)}$

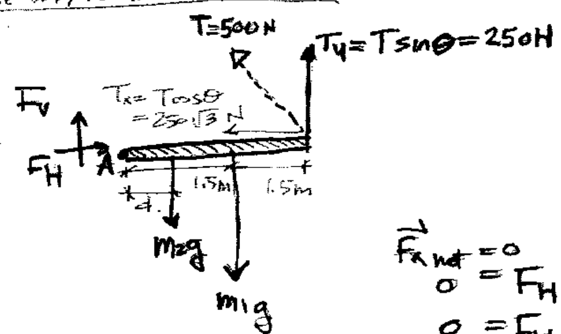
3) A horizontal bar with mass **m1 = 20 kg** and length **L = 3 m** is hinged at one end to a vertical wall, and also held to the wall, at its other end, by a thin, massless wire that makes an angle **θ = 30°** with the horizontal. A mass **m2 = 30 kg** is placed on the bar.



a) If the wire can withstand a maximum tension **T = 500 N**, what is the maximum **distance d** that mass **m2** can be placed on the bar, away from the wall?

b) What are the **horizontal F_H and vertical F_V components** of the force on the bar from the hinge, when the tension in the wire is **500 N**?

FREE BODY DIAGRAM OF THE BAR



In Equilibrium:

$\vec{F}_{x\,net} = 0$

$\vec{F}_{y\,net} = 0$

$\vec{\tau}_{net} = 0.$

$\vec{F}_{x\,net} = 0$

$0 = F_H + T_x$

$0 = F_H - 250\sqrt{3}\,N$

$$\boxed{F_H = +250\sqrt{3}\,N\ (\rightarrow)\ (433.0\,N)}$$

$\vec{F}_{y\,net} = 0$

$0 = -m_2g - m_1g + F_V + T_y$

$0 = -300N - 200N + F_V + 250N$

$$\boxed{F_V = 250\,N\ (\uparrow)}$$

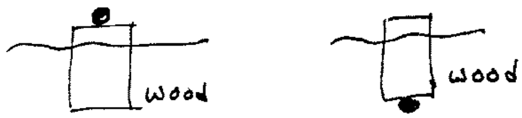
$\vec{\tau}_{net,A} = 0$

$0 = \tau_{F_V} + \tau_{F_H} + \tau_{m_2g} + \tau_{m_1g} + \tau_{T_y} + \tau_{T_x}$

$= 0 + 0 - (300N \cdot d) - (200N \cdot 1.5m) + (250N \cdot 3m) + 0$

$$\boxed{d = 1.5m}$$

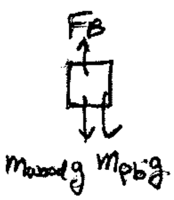
4) A block of wood with mass **m = 3.67 kg** has a density ρ_{wood} **= 6.00 x 10^2 kg/m^3**. It is to be loaded with lead, with density ρ_{Pb}**= 1.13 x 10^4 kg/m^3**, so that the wood will float in water with **90%** of its volume submerged below the level of the water.



a) If the lead is attached to the top of the wood, what mass of lead is needed?

b) If the lead is attached to the bottom of the wood, what mass of lead is needed?

a) $V_{wood} = \frac{m_{wood}}{\rho_{wood}}$



$$F_B - m_{wood}\cdot g - m_{Pb}\cdot g = 0$$

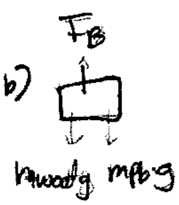
$$0.9(V_{wood})\cdot\rho_{water}\cdot\cancel{g} - m_{wood}\cancel{g} - m_{Pb}\cdot\cancel{g} = 0$$

$$0.9\left(\frac{m_{wood}}{\rho_{wood}}\right)\rho_{water} - m_{wood} = m_{Pb}$$

$$0.9\left(\frac{3.67\,kg}{600\,kg/m^3}\right)(1000\,kg/m^3) - 3.67\,kg = m_{Pb}$$

$$\boxed{1.835\,kg = m_{Pb}}$$

b)



$$F_b - m_{wood}\cdot g + m_{Pb}\cdot g = 0$$

$$(0.9(V_{wood}) + V_{Pb})\rho_{water}\cdot\cancel{g} - m_{wood}\cdot\cancel{g} - m_{Pb}\cdot\cancel{g} = 0$$

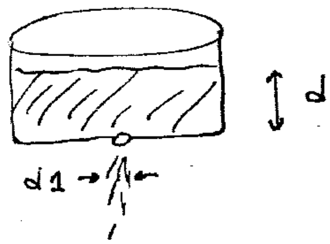
$$\left(0.9\frac{m_{wood}}{\rho_{wood}} + \frac{m_{Pb}}{\rho_{Pb}}\right)\rho_{water} - m_{wood} = m_{Pb}$$

$$0.9\frac{m_{wood}}{\rho_{wood}}\rho_{water} + m_{Pb}\frac{\rho_{water}}{\rho_{Pb}} - m_{wood} = m_{Pb}$$

$$\frac{0.9\frac{m_{wood}}{\rho_{wood}}\rho_{water} - m_{wood}}{(1 - \rho_{water}/\rho_{Pb})} = m_{Pb}$$

$$\boxed{2.013\,kg = m_{Pb}}$$

5) A large diameter tank on a tower is filled with water to a depth **d = 1 m**. The tank has a hole in the bottom with diameter $\mathbf{d_1 = 2\ cm}$.



a) What is the volume of water that flows out of the hole in a time of **10 s**?

a) If the water flows straight down from the bottom of the tank, at what distance from the bottom of the tank does the diameter of the water stream equal **1 cm**?

a) Bernoulli's Eq.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 = P_2 = P_{atm},\quad y_1 = d,\ y_2 = 0,\ V_1 \cong 0$$

$$\cancel{P_{atm}} + 0 + \rho_{water} \cdot g \cdot d = \cancel{P_{atm}} + \frac{1}{2}\rho_{water} V_2^2 + 0$$

$$\sqrt{2gd} = V_2$$

$$\sqrt{20}\ m/s. = V_2$$

Volume = Volume flow × time

$$= A_2 V_2\ t = (0.02\,m)^2 \pi \cdot \sqrt{20}\ m/s \cdot 10\,sec.$$

$$\boxed{= 0.0562\ m^3}$$

b) Continuity $A_1 V_1 = A_2 V_2$

$$(0.02\,m)^2 \pi \sqrt{20\,m/s} = (0.01\,m)^2 \pi\ V_2$$

$$V_2 = 4\sqrt{20}\ m/s$$

using kinematics, $V^2 = V_0^2 + 2a(y - y_0)$

$$(4\sqrt{20}\,m/s)^2 = (\sqrt{20}\,m/s)^2 - 2 \cdot 10\,m/s^2 (\Delta y)$$

$$\Delta y = -15\,m.$$

$\boxed{\text{ans: } 15\,m \text{ below the hole.}}$