University of California at Berkeley
Department of Physics
Physics 8A, Spring 2003

Second Midterm Exam April 9, 2003

You will be given 120 minutes to work this exam. No books, but you may use a handwritten note sheet no larger than an 8 1/2 by 11 sheet of paper. Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Write your answers directly on the exam, and if you have to use the back of a sheet make sure to put a note on the front. Do not use a blue book or scratch paper.

sin 45° = 0.707, cos 45° = 0.707, sin 30° = 0.500, cos 30° = 0.866

Rotational Inertias for radius R or length L:
sphere about axis: $(2/5)MR^2$ spherical shell about axis: $(2/3)MR^2$
disk about axis: $(1/2)MR^2$ hoop about axis: MR^2
rod about perpendicular at midpoint: $ML^2/12$

$$\frac{1}{2}\rho v^2 + yg\rho + P = \text{constant} \qquad \omega = \sqrt{\frac{k}{m}} \quad \omega = \sqrt{\frac{g}{l}} \quad \sum \vec{F} = m\vec{a}$$

Each part is worth the number of points indicated. These should sum to 100 points. Setup and explanation are worth almost all the of the points. Clearly state what you are doing and why. In particular, make sure that you explain what principles and conservation rules you are applying, and how they relate.

NAME: Lecture 1

SID NUMBER: – Solutions –

DISCUSSION SECTION NUMBER:________

DISCUSSION SECTION DATE/TIME: ________

1	
2	
3	
4	
5	
6	
total	110 points

Read the problems carefully.
Try to do all the problems.
If you get stuck, go on to the next problem.
Don't give up! Try to remain relaxed and work steadily.

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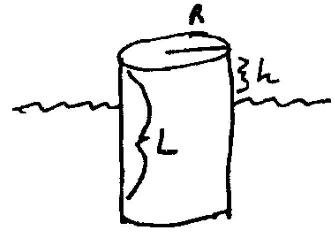
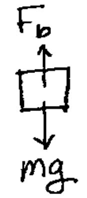
1) (25 points) A cylindrical block of wood with radius R, total length L and density ρ_b is floating in a liquid of density ρ_f. See figure.

a) Find the height of the block above the surface. This is marked as h in the figure.

b) The block is lifted a small distance x and released ("Small" means that x<<h). The block then bobs up and down. How long does it take to go through one complete cycle?



a)



$F_b = R^2\pi(L-h)\rho_f \cdot g$

$mg = R^2\pi(L)\rho_b \cdot g$

$\cancel{R^2\pi}(L-h)\rho_f \cdot \cancel{g} - \cancel{R^2\pi}(L)\rho_b \cdot \cancel{g} = 0$

$F_b - mg = 0.$

$(L-h)\rho_f - L\rho_b = 0$

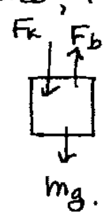
$L\rho_f - h\rho_f - L\rho_b = 0$

$\boxed{\dfrac{L(\rho_f - \rho_b)}{\rho_f} = h}$

b) In simple harmonic motion, $T = 2\pi/\omega$, $\omega = \sqrt{k/m}$

where $m = R^2\pi L \rho_b$ and k = force required to displace the system 1 unit amount (1 m).

To find k, let's suppose that the block moves 1 unit amount after a force, F_k, has been applied:

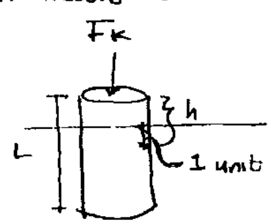


Then: $F_b - F_k - mg = 0$

$F_b = R^2\pi(L-h+1\text{ unit})\rho_f \cdot g$

$mg = R^2\pi(L)\rho_b \cdot g$,

From previous, $mg = R^2\pi(L-h)\rho_f \cdot g$.



So: $R^2\pi(L-h+1\text{unit})\rho_f \cdot g - F_k - R^2\pi(L-h)\rho_f \cdot g = 0.$

$R^2\pi(1\text{ unit})\rho_f \cdot g. = F_k$

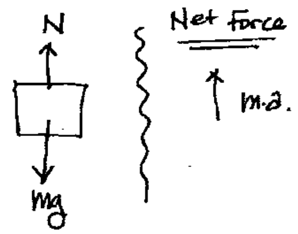
So $k = F_k/1\text{unit} = R^2\pi\rho_f \cdot g\, \cancel{(1\text{unit})}/\cancel{(1\text{unit})} = R^2\pi\rho_f \cdot g$

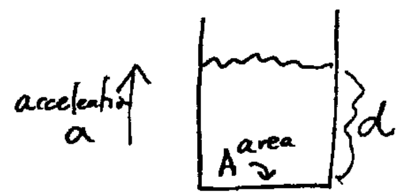
$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\dfrac{R^2\pi(L)\rho_b}{R^2\pi(\rho_f)g}} = \boxed{2\pi\sqrt{\dfrac{L\rho_b}{\rho_f g}}}$

25

2) ([illegible] points) A cup of liquid is on an elevator that's accelerating upward with acceleration a. The area of the cup is A, and the depth of the liquid is d. The liquid has a density of ρ_f. Use g for the acceleration of gravity. What is the pressure at the bottom of the cup? (Hint: think about the forces on liquid at the bottom of the cup) Make sure your explanation is clear.

Free body diagram.





$N - mg = m \cdot a$

$N = m(g+a) = \rho_f \cdot A \cdot d\,(a+g) =$ Force on fluid from bottom

Pressure:

~~$P = P_0 + \rho g h$~~

~~$P = (A \cdot d\, \rho_f)(g+a)(d)$~~

~~$\boxed{P = A d^2 \rho_f g + A d^2 \rho_f a}$~~

By Newton's 3rd law, the force on the bottom from the fluid is

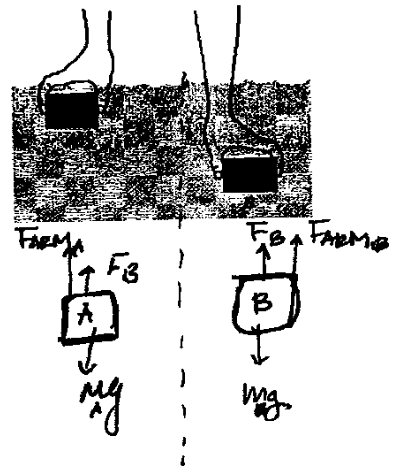
$F_{bf} = P \cdot A = \rho_f A d\,(g+a)$

$$\boxed{P = \rho_f \cdot d\,(g+a)}$$

This is the gauge pressure since i ignored atmosphere pressure above. The total pressure is

$$\boxed{P = P_{ATM} + \rho_f d\,(g+a)}$$

3) (10 points) Imagine holding two bricks under water. They would sink if you let them go. Brick A is just beneath the surface of the water, while brick B is at greater depth. Is the force needed to hold brick B in place larger, the same as, or smaller than the force needed to hold brick A? Make sure to explain your answer.



Ignoring the volume of the arms,

For Block A: $F_B - m_A g + F_{ARM A} = 0$

For Block B: $F_B - m_B g + F_{ARM B} = 0.$

If Brick A = Brick B; $m_A = m_B$, and $V_A = V_B$.

Then:

For Block A:

$$F_{ARM A} = m_A g - F_B.$$

$$F_{ARM A} = m_A g - V_A \rho_{water} g.$$

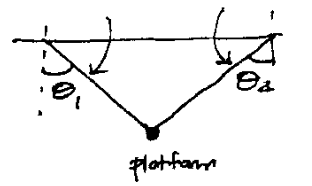
For Block B:

$$F_{ARM B} = m_B g - F_B$$

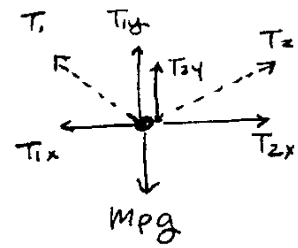
$$F_{ARM B} = m_B g - V_B \rho_{water} g$$

$$\boxed{F_{ARM A} = F_{ARM B}}$$

4) (20 points) A work platform is hung from the new Bay Bridge by two cables. The platform is positioned by shortening and lengthening the cables, so the cables may not be the same length. One cable has tension T_1 and makes an angle of θ_1 with the vertical. The other has tension T_2 and makes an angle of θ_2 with the vertical (see figure). Find an expression for T_1, the tension in cable 1, that does not depend on T_2.



Free Body Diagram of the platform.



$T_{1x} = -T_1 \sin\theta_1 \qquad T_{2x} = T_2 \sin\theta_2$

$T_{1y} = T_1 \cos\theta_1 \qquad T_{2y} = T_2 \cos\theta_2$

In equilibrium, $\left[\begin{array}{l} \vec{F}_{x\,net} = 0 \\ \vec{F}_{y\,net} = 0 \end{array}\right.$
$\vec{\tau}_{net} \doteq 0$

$\vec{F}_{x\,net} = 0$

$0 = T_{1x} + T_{2x} = 0$

$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0$

$T_2 = T_1 \dfrac{\sin\theta_1}{\sin\theta_2}$

$\vec{F}_{y\,net} = 0$

$0 = -M_p g + T_{1y} + T_{2y}$

$0 = -M_p g + T_1 \cos\theta_1 + T_2 \cos\theta_2$

$T_1 = \dfrac{M_p g}{\cos\theta_1} - T_2 \dfrac{\cos\theta_2}{\cos\theta_1}$

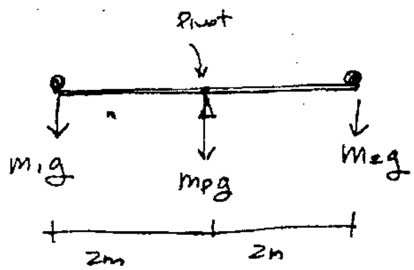
$T_1 = \dfrac{M_p g}{\cos\theta_1} - \left(T_1 \dfrac{\sin\theta_1}{\sin\theta_2}\right)\left(\dfrac{\cos\theta_2}{\cos\theta_1}\right)$

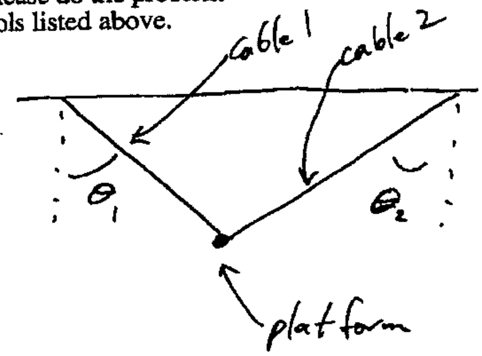
$T_1 = \dfrac{M_p g}{\cos\theta_1} - T_1 \dfrac{\sin\theta_1 \cos\theta_2}{\sin\theta_2 \cos\theta_1}$

$T_1 = \dfrac{M_p g}{\cos\theta_1 \left(1 + \dfrac{\sin\theta_1 \cos\theta_2}{\sin\theta_2 \cos\theta_1}\right)}$

$$\boxed{T_1 = \frac{M_p g}{\left(\cos\theta_1 + \dfrac{\sin\theta_1 \cos\theta_2}{\sin\theta_2}\right)}}$$

5) (20 points) Two point-like children of mass m_1=20kg and m_2=25kg are each sitting L=2 meters from the center of a see-saw (see figure). The mass of the see-saw plank is M_p=60kg. Use 10 m/s² for g. At the moment shown in the figure, what is the angular acceleration of the see-saw? Specify magnitude and direction. Please do the problem symbolically before plugging in any numbers, and use the symbols listed above.





$$\tau_{net} = (\vec{F}\cdot d)_{net} = I_{net}\,\alpha.$$

$$(\vec{F}.d)_{net} = +(m_1 g\cdot 2m) + (m_P g\cdot 0) - (m_2 g\cdot 2m)$$

$$= 400\,N\cdot m + 0 - 500\,N\cdot m$$

$$(\vec{F}.d)_{net} = -100\,N\cdot m \quad (\text{net clockwise torque})$$

$$I_{net} = I_{Rod} + I_{m_1} + I_{m_2}$$

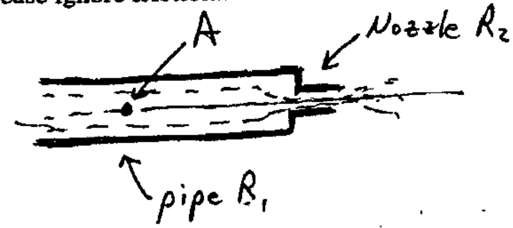
$$\left.\begin{aligned} I_{ROD} &= mL^2/12 = 60kg\cdot(4m)^2/12 \\ &= 80\,kg\cdot m^2 \\ I_{m_1} &= mR^2 = 20kg\cdot(2m)^2 \\ &= 80\,kg\cdot m^2 \\ I_{m_2} &= mR^2 = 25kg\,(2m)^2 \\ &= 100\,kg\cdot m^2. \end{aligned}\right\} \begin{aligned} I_{net} &= 80+80+100 \\ &= 260\,kg\cdot m^2. \end{aligned}$$

$$(\vec{F}.d)_{net} = I_{net}\,\alpha.$$

$$-100\,N\cdot m = 260\,kg\,m^2\,\alpha$$

$$\boxed{\begin{aligned} \alpha &= -\tfrac{5}{13}\ \text{rad}/s^2 \\ &(-0.385\ \text{rad}/s^2) \\ &\text{net clockwise ang accel.} \end{aligned}}$$

6) (10 points) Water is moving through a pipe of radius R_1 until it comes to a nozzle of radius R_2. It passes through the nozzle at velocity v_2 and exits at atmospheric pressure. What is the pressure in the pipe at point A in the figure? Please ignore frictional losses in the fluid.



Continuity:

$$A_1 V_1 = A_2 V_2$$
$$(\pi R_1^2) V_1 = (\pi R_2^2) V_2$$
$$V_1 = \frac{R_2^2}{R_1^2} V_2$$

Bernoulli's Eq.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

$y_1 = y_2 = 0$, $V_1 = \frac{R_2^2}{R_1^2} V_2$ (from previous), $P_2 = P_{atm}$

$$P_1 + \frac{1}{2}\rho_{water}\left(\frac{R_2^2}{R_1^2} V_2\right)^2 + 0 = P_{atm} + \frac{1}{2}\rho_{water} V_2^2 + 0$$

$$P_1 = P_{atm} + \frac{1}{2}\rho_{water} V_2^2 - \frac{1}{2}\rho_{water}\frac{R_2^4}{R_1^4} V_2^2$$

$$\boxed{P_1 = P_{atm} + \frac{1}{2}\rho_{water} V_2^2 \left[1 - \frac{R_2^4}{R_1^4}\right]}$$

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