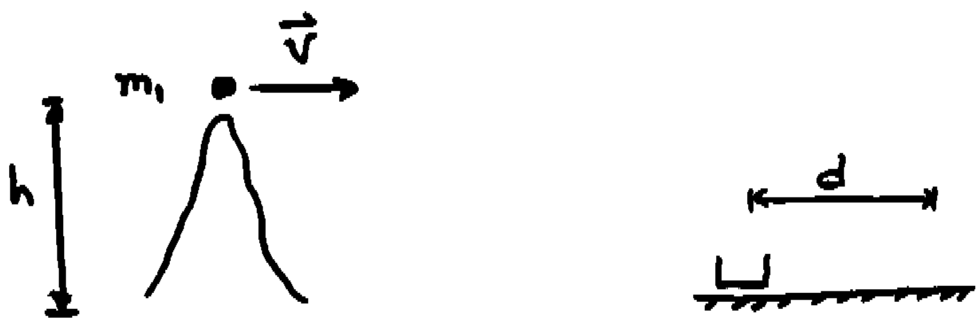
1) A ball with mass $\mathbf{m_1 = 2\ kg}$ is thrown horizontally, off the top of a mountain at a height $\mathbf{h = 100\ m}$ above the ground, with an initial velocity $\mathbf{v = 20\ m/s}$. The ball lands in a box with mass $\mathbf{m_2 = 4\ kg}$ on the ground in an inelastic collision. The vertical component of the ball's velocity is absorbed by cushioning in the box, but the box slides horizontally, with uniform deceleration over a distance $\mathbf{d = 10\ m}$ until stopped.



a) How long is the ball in the air?

b) How far does the ball travel horizontally in the air?

c) What is the kinetic energy of the ball as it hits the box?

d) What is the coefficient of friction of the box with the ground?

a) $h = \frac{1}{2} g t^2 \qquad t = \sqrt{20} = \boxed{4.5\ s}$

b) $\ell = v t = \boxed{90\ m}$

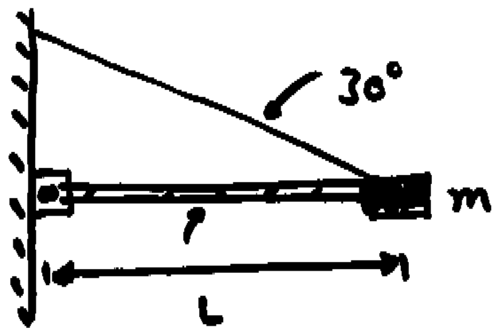
c) $E = mgh + \frac{1}{2} m v^2 = \boxed{2400\ J}$

d) $m_1 v = (m_1 + m_2) v'$

$\frac{1}{2}(m_1 + m_2)(v')^2 = \mu (m_1 + m_2) g d$

$\boxed{\mu = 0.22}$

2) A mass **m = 50 kg** is placed at the end of a massless rod (of length **L = 2 m**). The rod is supported by a massless cable (extending from the wall, connected at the end of the rod, and at an **angle of 30°** to the horizontal) and by a pivot point (at the wall).

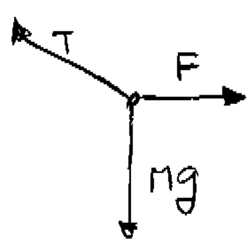


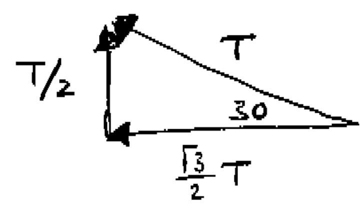
a) What is the tension in the cable?

b) What is the magnitude and direction of the force exerted by the wall on the rod at the pivot point?

c) If the **rod** is broken and the mass swings into the wall, and then it stops in a time **t = 0.1 s**, what is the force exerted by the wall on the mass?

a)



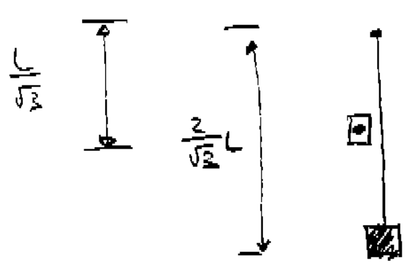


$$T = 2mg = \boxed{1000\ N}$$

b) direction = away from wall

$$F = \frac{\sqrt{3}}{2} T = \sqrt{3}\, mg = \boxed{866\ N}$$

c) L = 2m



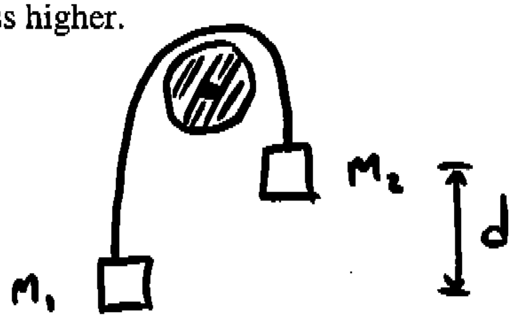
$$mg\left[\frac{2}{\sqrt{3}}L - \frac{1}{\sqrt{3}}L\right] = \frac{1}{2} m v^2$$

$$g\left[\frac{2}{\sqrt{3}}\right] = \frac{v^2}{2}$$

$$F = \frac{\Delta p}{t} = \frac{mv}{t} = \boxed{2400\ N}$$

4

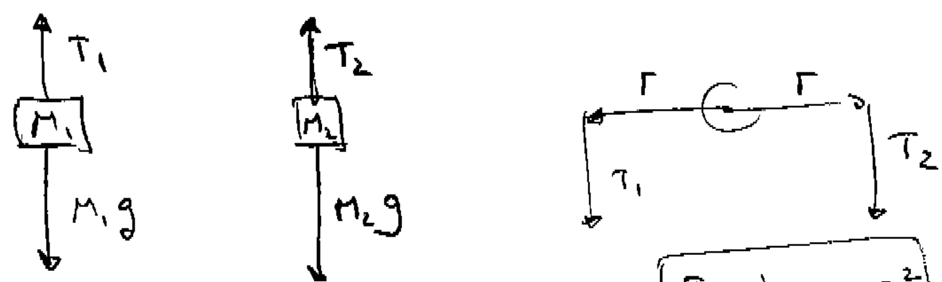
3) Two masses $\mathbf{m_1 = 20\ kg}$ and $\mathbf{m_2 = 30\ kg}$ are suspended by a massless rope. The rope lies over a pulley disk with mass $\mathbf{m_p = 25\ kg}$ (uniformly distributed) and **radius r = 15 cm**. Initially the masses are separated by a vertical distance **d = 2 m**, with the heavier mass higher.



a) As the masses are released, what are the **tensions** in the rope supporting the two masses?

b) How long does it take for the masses to pass each other, at the same vertical height?

a)



$$\boxed{I = \tfrac{1}{2} M_p r^2}$$

$$\left.\begin{aligned} T_1 - M_1 g &= M_1 a \\ T_2 - M_2 g &= -M_2 a \\ -rT_1 + rT_2 &= I\alpha = I\frac{a}{r} \end{aligned}\right]$$

$$a = \left[\frac{M_2 - M_1}{M_1 + M_2 + M_p/2}\right] g$$

$$a = 1.6\ \mathrm{m/s^2}$$

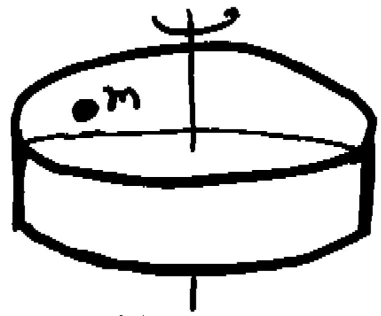
$$T_1 = M_1 (g + a) = \boxed{232\ \mathrm{N}}$$

$$T_2 = M_2 (g - a) = \boxed{252\ \mathrm{N}}$$

b) $$d/2 = \tfrac{1}{2} a t^2$$

$$t = \sqrt{d/a} = \boxed{1.8\ \mathrm{s}}$$

4) A mass **m = 10 kg** is positioned against the inside wall of a rotating cylinder with radius **r = 2 m**. The coefficient of friction between the cylinder and the mass is **μ = 0.1**.



a) How many rotations per second (minimum) must the cylinder make to keep the mass from falling down the side of the cylinder?

b) What is the ratio of the force on the mass under conditions of part (a) to the usual force used by a person to support the mass?

(a)

$\mu \frac{mv^2}{r}$

mg

$$\frac{\mu m v^2}{r} = mg$$

$$\Rightarrow \quad v = \sqrt{\frac{rg}{\mu}}$$

$$vT = 2\pi r$$

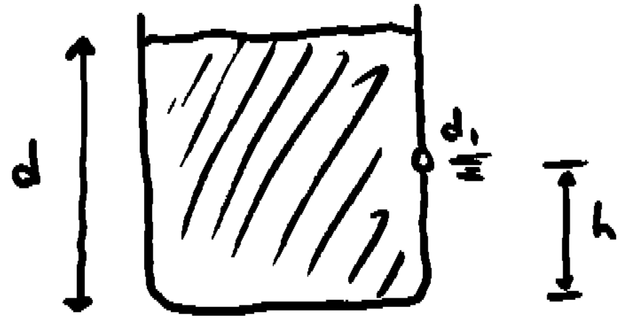
$$\omega = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{\mu r}} = \boxed{1.1 \text{ rot/s}}$$

(b)

$$\frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg} = \frac{1}{\mu} = \boxed{10}$$

6

5) A very large diameter water tank is filled with water to a depth **d = 10 m**. The tank has a hole in the side at a height **h = 5 m** with diameter **d_1 = 1 cm**.



a) <u>What is the volume of water that flows out of the hole in a time of 5 s?</u>

b) <u>How far from the base of the water tank does the stream of water from the hole hit the ground?</u>

a) $\cancel{P} + \cancel{\frac{1}{2}\rho v_1^2} + \rho g y_1 = \cancel{P} + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$$\Rightarrow \cancel{\rho} g d = \frac{1}{2}\cancel{\rho} v_2^2 + \cancel{\rho} g h$$

$$v = \sqrt{2g(d-h)}$$

$$Vol = vAt = \sqrt{2g(d-h)}\,\frac{\pi d_1^2}{4}\,t$$

$$= \boxed{3.9 \times 10^{-3}\ M^3}$$

b) $h = \frac{1}{2} g t^2$

$$t = \sqrt{2h/g}$$

$$s = vt = \sqrt{2g(d-h)}\,\sqrt{2h/g}$$

$$= 2\sqrt{(d-h)(h)}$$

$$= \boxed{10\ M}$$

7

6) An ice cube with mass **m = 100 g** is at a temperature $\mathbf{T_{ice}}$**= 0 °C** and held on a cloth in a person's hand in a room at **0 °C**. The contact area between the surface of the ice cube and person's hand is **A = 25 cm²** The thermal conductivity of the cloth is **k = 1 W/m/K** and its thickness is **t = 0.1 cm**. The person's temperature is constant at $\mathbf{T_{person}}$ **= 37 °C**.

a) How long does it take the ice cube to melt?

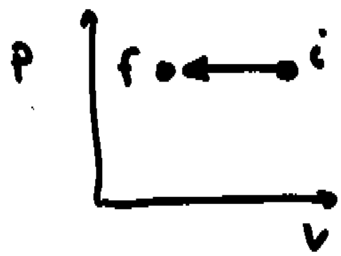
$$P = \frac{kA(T_1 - T_2)}{L} = \frac{(1)(25\ 10^{-4})(37)}{10^{-3}} = 93\ W$$

require $\quad Pt = M\ 330\ kJ/kg = 10^{-1}(330) = 33\ kJ$

$$t = \frac{33\ 10^3}{93} = \boxed{350\ s}$$

8

7) One mole of an ideal gas at a pressure **P = 2 atmospheres** and temperature **T = 100K** is compressed from its **initial volume**, to **one-third of that volume**, at constant pressure.



a) What is the new temperature of the gas?

b) What is the work done by the gas during the compression?

c) What is the heat energy input during the compression?

d) What is the change in entropy of the gas?

c) $\frac{PV}{T} = \frac{PV/3}{T'}$ $T' = \frac{100}{3} = \boxed{33k}$

b) $W = P\Delta V = P(V/_2 - V) = -PV/_2$

$-\frac{PV}{2} = -\frac{nRT}{2} = \boxed{550\ J}$

c) $\Delta E = Q - W$

$= n C_v \Delta T$

$\boxed{Q = -1400\ J}$

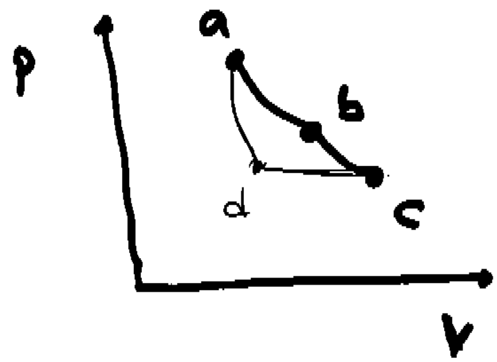
d) $dQ = n C_p \Delta T$

$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{n C_p dt}{T} = n C_p \ln (T_f/T_i)$

$= \boxed{-23\ J}$

9

8) In step (a – b) of a thermodynamic process, one mole of an ideal gas undergoes an isothermal expansion at temperature $\mathbf{T_1 = 300K}$ with work done $\mathbf{W_1 = 2 \times 10^3}$ **J.** In step (b – c), the gas undergoes an adiabatic expansion in which the work done is $\mathbf{W_2 = 1.5 \times 10^3}$ **J.**



a) What is the heat energy added to the gas in step (a – b)?

b) What is the new temperature after step (b – c)?

c) If steps (a – b), and then (b – c) are the first two parts of a Carnot cycle, draw the cycle in the figure; what is the efficiency of the cycle?

a) $\Delta E = Q - W_1 = 0$

$Q = W_1 = \boxed{2 \times 10^3 \text{ J}}$

b) $\Delta E = Q - W_2$

$Q = 0 \qquad \Delta E = -W$

$\Delta E = n C_V \Delta T$

$\frac{3}{2} n R (T_f - T_i) = -W_2 = -1.5 \times 10^3 \text{ J}$

$\Rightarrow \boxed{T_f = 180 \text{ K}}$

c) c → d isothermal
d → a adiabatic

$\varepsilon = \frac{W_{NET}}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} = 1 - \frac{180}{300}$ $\boxed{40\%}$ 10