Physics 8A Section 2 Midterm # 2

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Your name

November 7, 2002

210 (Shahed) FRi 11-12pm
Discussion session number / GSI name

1) DON'T OPEN THIS EXAM UNTIL INSTRUCTED TO BEGIN
2) Sit one seat away from anyone else.
3) Do all your work on the page (front/back) indicated for each problem.
4) Show all work; don't just write an answer without showing your reasoning.
5) This is a closed book exam but calculators are allowed.
6) To simplify the math, take the acceleration due to gravity as $g = 10$ m/s^2.
7) Possibly useful equations include:

$F = dp / dt = m a$
$p = mv$
$F_c = m v^2 / r$
$x = x_0 + v_0 t + 1/2\, a t^2$
$W = F x$
$U = m g h$
$K = 1/2\, m v^2$
$\tau = I \alpha$
$I = mr^2$
$\tau = r \times F = dL / dt$
$L = I \omega$
$v = \omega r$
$a = \alpha r$

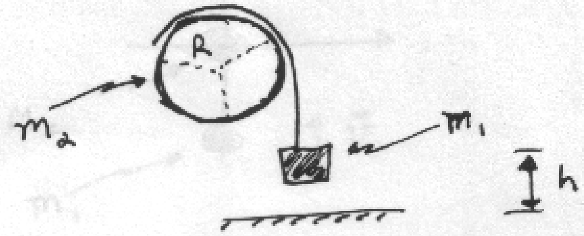
$I = I_{com} + mh^2$
$K = 1/2\, I \omega^2$
$F/A = E\, \delta L/L$
$F = G M_e m / r^2$
$F = -\, dU / dx$
$G = 7 \times 10^{-11}$ N m^2 / kg^2
$V_{escape} = (2 G M_e / R_e)^{1/2}$
$M_{earth} = 6 \times 10^{24}$ kg
$R_{earth} = 6 \times 10^6$ m
$v_1 A_1 = v_2 A_2$
$p + 1/2\, \rho v^2 + \rho g y =$ constant
1 atmosphere $= 10^5$ Pa
$\rho (H_2O) = 1$ g / cc

SCORING - we'll handle this space :)

1) 20
2) 20
3) [redacted]
4) [redacted]
5) [redacted]

TOTAL [redacted]

1) A mass $\mathbf{m_1 = 10\ kg}$ hangs at a distance $\mathbf{h = 3\ m}$ above the floor from a massless rope that is wrapped around a frictionless pulley. The pulley's mass $\mathbf{m_2 = 20\ kg}$ is concentrated at its rim, which has a radius $\mathbf{R = 2\ m}$.



a) What is the time that it takes for the mass to fall to the floor when it is released?

b) What is the kinetic energy of the mass $\mathbf{m_1}$ when it hits the floor?

c) What is the rotation speed of the pulley, when the mass hits the floor? right before

$t = \sqrt{\frac{\ell}{a}}$ $m_1 = 10kg$ $\Delta h = 3m$

$m_2 = 20kg$

$R = 2m$

E initial

$U = (10)(g)(3m)$

$= 294\ J$

E final

$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$= \frac{1}{2}\ m r^2 \frac{v^2}{r^2} + \frac{1}{2}m_1 v^2$

$10(9.8)(3) = \frac{1}{2}(20)(2^2)\left(\frac{v^2}{2^2}\right) + \frac{1}{2}(10)v^2$

$294 = 10v^2 + 5v^2$

$294 = 15v^2$

$v^2 = 19.6$

$v = 4.43 m/s$

a) $\Delta y = v_{0y}t - \frac{1}{2}gt^2$

$-3m = (4.43m/s)t - \frac{1}{2}(9.8)t^2$

$-3 = 4.43t - 4.9t^2$

$4.9t^2 - 4.43t - 3 = 0$

$t = 1.36s$

b) $KE = \frac{1}{2}m_1v^2 = \frac{1}{2}(10)(4.43)^2$

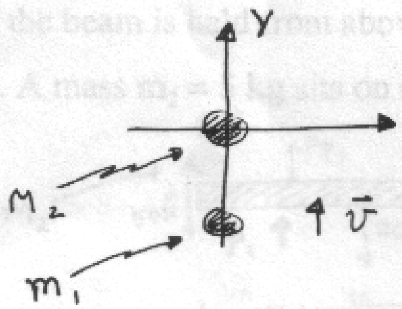
$= 98.1J$

c) $\omega r = v$

$\omega = \frac{v}{r} = \frac{4.43m/s}{2m} = 2.2\ rad/s$

(20)

2) A mass **m = 4 kg** slides without friction on a table, in a direction along the y-axis and with a speed **v = 2 m/s**. It collides with a stationary mass **m = 4 kg**.



a) If the two masses stick together, what is the resulting speed and direction of the two masses after the collision? inelastic → conserve momentum

b) If the two masses collide elastically, and the mass m_2 moves along the y-axis, what is the resulting speed and direction of mass m_1 after the collision? → conserve KE & p

$m_1 = 4kg$

$v_1 = 2 m/s$ (y)

$m_2 = 4kg$

$v_2 = 0 m/s$

a) $m_1v_1 + m_2v_2 = (m_1+m_2)v_f$

$(4)(2) + (4)(0) = (4+4)v_f$

$v_f = \frac{8}{8} = $ 1 m/s   in the y direction

b) elastic

momentum: $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$4(2) + 0 = 4(v_{1f}) + 4(v_{2f})$

$8 = 4(v_{1f} + v_{2f})$

$v_{1f} + v_{2f} = 2$ $v_{2f} = 2 - v_{1f}$

KE! $\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$ $(m_1 = m_2)$

$(2m/s)^2$ 0

$2^2 = v_{1f}^2 + v_{2f}^2$

$4 = v_{1f}^2 + v_{2f}^2$ $v_{2f} = 2 - v_{1f}$

$4 = v_{1f}^2 + (2-v_{1f})^2$ $(2-v_{1f})(2-v_{1f}) = 4 - 2v_{1f} - 2v_{1f} + v_{1f}^2$

$4 = v_{1f}^2 + 4 - 4v_{1f} + v_{1f}^2$

$0 = 2v_{1f}^2 - 4v_{1f}$

$0 = 2v_{1f}(v_{1f} - 2)$

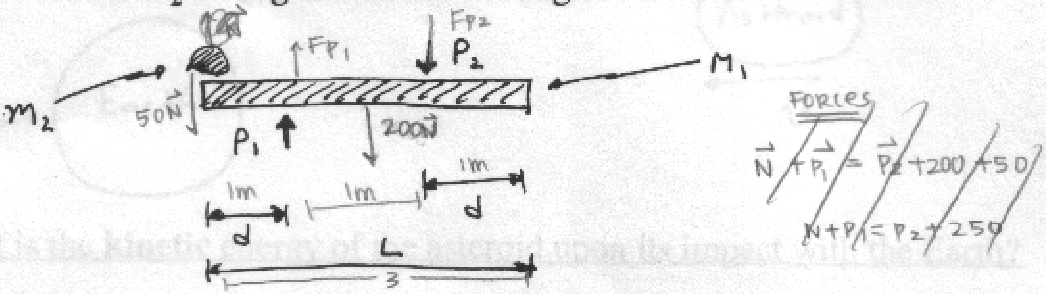
$v_{1f} = 0 m/s$ or $2 m/s$ b/c $v_{2f} = 2 - v_{1f}$

$v_{2f} \neq 0$ b/c it moves

$v_{2f} = 2 m/s$

$v_{1f} = 0 m/s$

3) A solid beam with mass $\mathbf{m_1 = 20\ kg}$ and length $\mathbf{L = 3\ m}$ is held at two points $\mathbf{P_1}$ and $\mathbf{P_2}$. At $\mathbf{P_1}$ the beam is held from below at a distance $\mathbf{d = 1\ m}$ from the left edge of the beam. At $\mathbf{P_2}$ the beam is held from above at a distance of $\mathbf{d = 1\ m}$ from the right edge of the beam. A mass $\mathbf{m_2 = 5\ kg}$ sits on the left edge of the beam.



a) <u>What is the direction and magnitude of the forces exerted at points</u> $\mathbf{P_1}$ and $\mathbf{P_2}$?

Torques (P_1)

$0 = P_1(0) + 50(1) - (200)(\frac{1}{2}) - P_2(1)$

$P_2 = -50\ N$

Torques (P_2)

$0 = (\frac{1}{2})(200) + (P_1)(1) + 50(2)$

$P_1 = (+) 200\ N$

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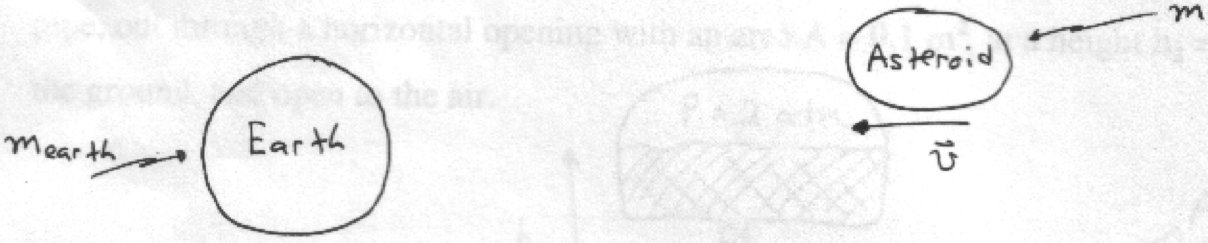
$P_1 = 200N$ clockwise
$P_2 = 50N$ clockwise

direction of a Force: Up or down

4) Earth is under attack from an alien civilization that shoots an asteroid at us from the edge of the universe. The asteroid has a mass **m = 100 kg** and an initial speed **v = 10 km/s.**



a) What is the **kinetic** energy of the asteroid upon its impact with the Earth?

b) Should this energy release affect the rotation of the Earth about its axis?

(hint: consider the rotational energy of the Earth, remembering that the moment of inertia of a solid sphere $I = 2/5\ mR^2$)

$m_a = 100\text{kg}$

$v_{ia} = 10\text{ km/s} = 10{,}000\text{ m/s}$

$m_e = 6\times10^{24}\text{ kg}$

$v_{ie} = 0\text{ m/s}$

a) $KE = 1/2\, mv^2$

$= 1/2\,(100)(10{,}000)^2$

2 $= \boxed{5\times10^9\text{ J}}$

b) ~~$F = GM_em/r^2 = (6.0\times10^{-11})(6\times10^{24})(100)/(6\times10^6 m)^2 = 1167\text{ N}$~~

b) conserve p

$(100)(10{,}000) + 0 = (6\times10^{24} + 100)\, v_f$

$v_f = 1.7\times10^{-19}\text{ m/s}$

$I_{earth} = 2/5\, mR^2$

$= \frac{2}{5}(6\times10^{24})(6\times10^6)^2$

$= 8.64\times10^{37}$ ✓

clockwise... (-)

Rotational Energy Earth $= 1/2\, I\omega^2$

$= 1/2\,(8.64\times10^{37})(1.07\times10^{-14})^2$

$= 5\times10^9\text{ J}$

$\omega = \frac{2\pi}{24\text{ hrs}}$

conserve KE (-

$\frac{1}{2}mv^2$ (asteroid) $- 1/2\, I\omega^2$ (earth) $= 1/2(m+M)v^2$ (system) $+ 1/2\,(m+M)(r^2)(\frac{v^2}{r^2})$ (system)

$-\frac{1}{2}(100)(10{,}000)^2 - 1/2\,\omega^2 = 1/2(6\times10^{24}+100)(1.7\times10^{-19})^2 + 1/2(6\times10^{24}+100)(1.7\times10^{-19})^2$

(8.64×10^{37})

$-5\times10^9 + 4.32\times10^{37}\omega^2 = 8.67\times10^{-14} + 8.67\times10^{-14}$

$4.32\times10^{37}\omega^2 = -5\times10^9$

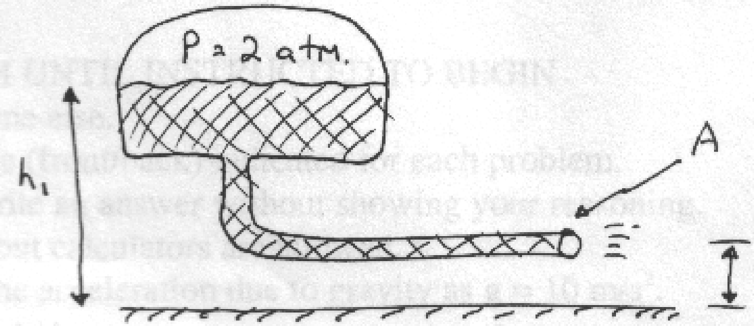
$\omega_{i\,Earth} = 1.07\times10^{-14}\text{ rad/s}$

3

yes it should affect the rotation b/c Earth's original energy was 5×10^9 J, and now it's being hit with the asteroid of the same energy.

5) Water is contained in a large diameter tank, which is sealed and held at a pressure **P = 2 atmospheres**. The top of the water in the tank is at a height **h_1 = 10 m** above the ground. The water flows through a hole in the bottom of the tank and through a frictionless pipe, out through a horizontal opening with an area **A = 0.1 m^2**, at a height **h_2 = 5 m** above the ground, and open to the air.

$P_t =$



<u>a) What is the horizontal distance from the end of the pipe that the water shoots out, before the water hits the ground?</u> (you can use helpful approximations)

$P_t = 2\,atm = 2\times10^5\,Pa$ ✓
$h_t = 10m$ ✓
$h_p = 5m$ ✓
$P_p = 1atm = 1\times10^5\,Pa$ ✓
$x = x_0 + v_{0x}t + \frac{1}{2}at^2$ ✓ (0)

$A_p = 0.1m^2$ $v_p = ?$
$A_t =$ large $v_t = ?$
$A_p v_p = A_t v_t$
$v_p = \frac{A_t}{A_p} v_t$ $(v_p \gg v_t)$ $v_t \approx 0$

$P_t + \frac{1}{2}\rho v_t^2 + \rho g h_t = P_p + \frac{1}{2}\rho v_p^2 + \rho g h_p$ (0)

$2\times10^5 + (998)(10)(10) = 1\times10^5 + \frac{1}{2}(998)(v_p^2) + (998)(10)(5)$ ✓

$299{,}800 = 149{,}900 + \frac{1}{2}(998)v_p^2$

$149{,}900 = \frac{1}{2}(998)v_p^2$

$v_p^2 = 300$ $v_p = 17.33\,m/s$ ✓

$\Delta y = v_{0y}t - \frac{1}{2}gt^2$
$-5 = (\cancel{17.33}\ 0)t - \frac{1}{2}(10)t^2$
$-5 = 17.33t - 4.9t^2$
$4.9t^2 - 17.33t - 5 = 0$ X
$t = 3.8s$

$x - x_0 = v_{0x}t$
$x - x_0 = (17.33)(3.8)$
$= \boxed{65.9m}$

(-2)