University of California at Berkeley
Department of Physics
Physics 8A, Fall 2007

Final Exam
Dec 15, 2007 5:00 PM

You will be given 170 minutes to work this exam. No books, but you may use a double-sided, handwritten note sheet no larger than an 8 1/2 by 11 sheet of paper. No electronics of any kind (calculator, cell phone, iPod, etc) are allowed.

Your description of the physics involved in a problem is worth significantly more than any numerical answer. Show all work, and take particular care to explain what you are doing. Write your answers directly on the exam, and if you have to use the back of a sheet make sure to put a note on the front. Do not use a blue book or scratch paper.

Each part is worth the number of points indicated. These should sum to 200 points. Setup and explanation are worth almost all of the points. Clearly state what you are doing and why. In particular, make sure that you explain what principles and conservation rules you are applying, and how they relate.

There are two pages of info at the back. You can tear them off and keep them separate if you'd like.

NAME: ______________________________________

SID NUMBER: ________________________________

DISCUSSION SECTION NUMBER: _________________

DISCUSSION SECTION DATE/TIME: _______________

1	
2	
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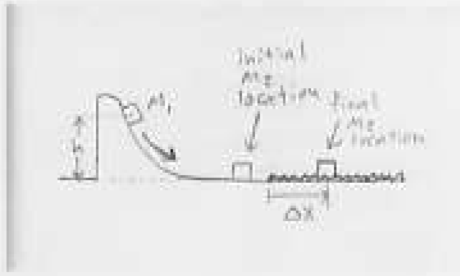
Total	

Read the problems carefully.
Try to do all the problems.
If you get stuck, go on to the next problem.
Don't give up! Try to remain relaxed and work steadily.

Problem 1 (30points) Sliding block



A block of mass M_1 = 1kg slides down a frictionless track that smoothly transitions into a horizontal table top. The block starts from rest at a height h = 5m above the table top. Once on the table top, the first block collides elastically with a second block of equal mass M_2 = 1kg. The side of the table top near the track is frictionless, but the table is textured on the other side so that the coefficient of kinetic friction is $\mu_k = 0.5$ on the textured part. You may treat the blocks as essentially point-like masses with no physical extent.

a) How fast is the first block moving when it reaches the table top?
b) After the collision, does the **first** block 1) continue moving in the same direction as it was before the collision, 2) bounce back in the opposite direction, or 3) is it not moving at all? Please explain your answer.
c) After the blocks collide, what is the speed, v_2, of the **second** block?
d) After the collision, when the second block slides into the textured region, what is the magnitude of the force of friction acting on the block?
e) How far, ΔX, does the second block slide into the textured part of the table before the block comes to rest?

a) $E_{total_i} = P.E._i + \cancel{K.E._i}^{\to 0} = M_1 g h$

energy is conserved $\Rightarrow$ $\cancel{P.E._f}^{\to 0} + K.E._f = M_1 g h = (1kg)(10\frac{m}{s^2})(5m) = 50j$

$K.E._f = \frac{1}{2} M_1 v_{1_f}^2 = 50j \Rightarrow v_{1_f} = \sqrt{100}\,\frac{m}{s} = \boxed{10\,\frac{m}{s}}$

b) elastic collision $\Rightarrow$ K.E. is conserved <u>and</u> momentum is conserved.

3 since $M_1 = M_2$, as seen from the center of mass, the two blocks come together at the same speed, $\frac{1}{2}v_i$; then bounce away at same speed. So, as seen by an observer standing beside the table, the C.O.M. is moving to the right at $\frac{1}{2}v_{1_i}$, and after the collision the 1st block is stationary $\left(\underset{\text{as seen in C.O.M. frame}}{v_{1f}} + \underset{\text{as seen in table frame}}{v_{C.O.M.}} = -\frac{1}{2}v_i + \frac{1}{2}v_i = 0\,\frac{m}{s}\right)$.

c) $\overset{\text{conc. of p.}}{v_{2_a}} = v_{1_b} = \boxed{10\,\frac{m}{s}}$

d) kinetic friction: $F_{fr} = \mu_k F_g = 0.5 \cdot M_2 g$
$= 0.5 \cdot 1kg \cdot 10\,\frac{m}{s^2}$
$= \boxed{5N}$

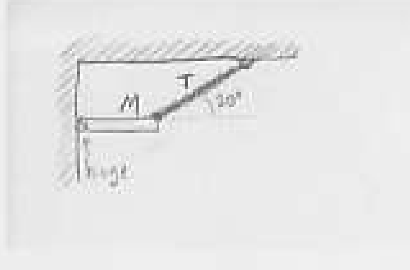
e) $a = \frac{F_{net}}{m} = 5\,\frac{m}{s^2}$

$v_f = v_i + at \Rightarrow t = \frac{v_f - v_i}{a} = \frac{0 - 10\,m/s}{-F_{fr}/m_2} = \frac{10\frac{m}{s}\,1kg}{5\,kg\frac{m}{s^2}} = 2s$

$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 10\frac{m}{s}\cdot 2s + \frac{1}{2}(-5\,\frac{m}{s^2})(2s)^2 = 0 + 20m - 10m$

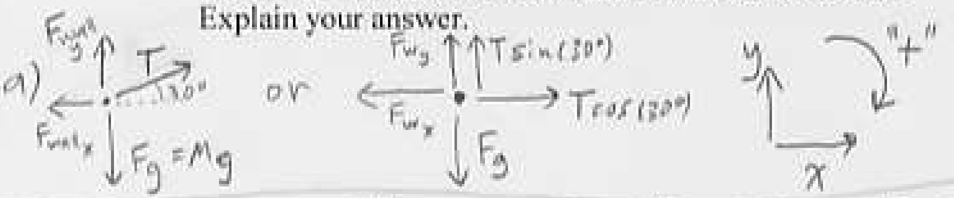
$\Delta x = x_f - x_i = x_f = \boxed{10m}$

Problem 2 (45 points) Rope and bar



A bar of mass $M = 10$kg and length $L = 1$m is connected to a hinge attached to a vertical wall so that the bar is free to rotate up or down in a vertical plane. The bar is suspended from the ceiling by a massless rope that is at an angle of 30° from the horizontal. The bar is not moving for parts a) through c). The moment of inertia of a bar rotating about one end is $I = ML^2/3$.

a) Draw a free body diagram for the bar. Please clearly label all forces.
b) What is the tension in the rope?
c) What is the *horizontal component* of the force exerted by the wall on the bar? If this force is nonzero, clearly indicate whether the force points to the left or to the right.
d) Now someone cuts the rope. Draw a free body diagram for the bar immediately after the rope is cut.
e) Immediately after the rope is cut, what is the *vertical component* of the force of the hinge on the bar? Indicate magnitude and direction.
f) Immediately after the rope is cut, what is the angular acceleration of the bar? Please indicate what convention you are using for the sign of clockwise and counterclockwise rotations.
g) What is the total kinetic energy of the bar (including rotational kinetic energy) immediately before it hits the wall at the bottom of its downward swing (see sketch)? Explain your answer.

a) 

b) N2L $\vec{F}_{net} = M\cancel{\vec{a}}^{\,0} \Rightarrow \begin{cases} x: F_{wx} + T\cos(30°) = 0 \Rightarrow F_{wx} = \frac{1}{2} \cdot T \\ y: F_{wy} + T\sin(30°) - Mg = 0 \end{cases}$

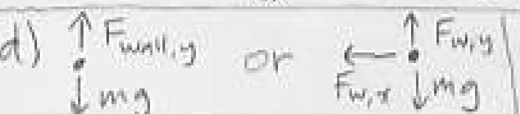
$N2L_\alpha$ $\tau_{net} = I\cancel{\alpha}^{\,0}$ I'll use pivot point at hinge:

$\tau_{hinge} + \tau_g + \tau_T = 0$

$0 \quad + \frac{L}{2}Mg - LT\sin(30°) = 0 \Rightarrow T = Mg = 10\text{kg} \cdot 10\text{m/s}^2 = \boxed{100N}$

c) Force from wall on bar is to the left:

N2L: $F_{wx} = -T\cos(30°) = -100N(.9) = \boxed{-90N}$ to the left

d) 

N2L: $\vec{F}_{net} = m\vec{a}$: y: $F_{w,y} - Mg = Ma_y$ — ①

$N2L_\alpha$: $\tau_{net} = I\alpha$: $\tau_g = I\alpha$

e) 

relate a and α:

$-a_y = \frac{L}{2}\alpha \Rightarrow \alpha = \frac{-2a_y}{L}$

$+\frac{L}{2}Mg = I\alpha = \frac{ML^2}{3}\alpha = \frac{-ML^2}{3}\frac{2}{L}a_y$

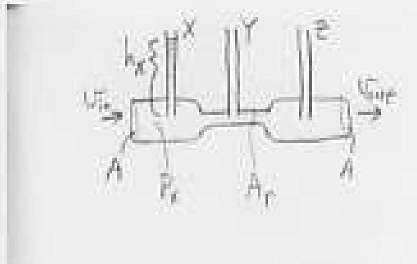
$a_y = -\frac{\cancel{L}\,3}{4\cancel{M}L^{\cancel{2}}}\cancel{M}g = -\frac{3}{4}g$ — ②

f) $\alpha = -\frac{2a_y}{L} = \frac{-2}{1\cancel{m}}\left(-\frac{30\cancel{m}}{4\,s^2}\right) = \boxed{+15\,s^{-2}}$ clockwise

① and ② $\Rightarrow$ $-Mg + F_{w,y} = -\frac{3}{4}Mg \Rightarrow F_{w,y} = +\frac{1}{4}Mg = \boxed{25N}$ upward

g) cons. of E: $K.E._{final} = P.E._i - P.E._f = \frac{L}{2}Mg = \frac{1m}{2}10\text{kg}\,10\frac{m}{s^2} = \boxed{50J}$

Problem 3 (35 points) Fluid flow through a pipe



Water is flowing through a horizontal pipe of cross-sectional area $A = 1\ m^2$, with a restriction in the middle with area $A_r = 0.25\ m^2$, as shown in the sketch. There are three vertical "stand pipes" inserted through the top of the horizontal pipe labeled X, Y and Z; water is not flowing up or down the stand pipes. For parts a) through d), assume that there are no viscous drag forces.

a) Water is moving with velocity $v_{in} = 1$ m/s into the left side of the horizontal pipe. How fast (v_{out}) is the water leaving the right side of the pipe? Please justify your answer.
b) What is the velocity ($v_{restr.}$) of the water in the restricted part of the pipe?
c) Remembering that the density of water is $\rho = 1000\ kg/m^3$, what is the pressure P_X of the water at the left end of the pipe if the height h_X of the water column in standpipe X is 1 m above the horizontal pipe?
d) How tall (h_Y) is the column of water in standpipe Y above the restricted part of the horizontal pipe?
e) If viscous drag forces are present, then will the water height (h_Z) in standpipe Z be greater than, less than, or equal to the water height (h_X) in standpipe X?
f) If viscous drag forces are present, will the velocity of water leaving the pipe (v_{out}) be less than, greater than, or equal to the velocity of water entering the pipe (v_{in})? Explain your answer.

a) water's incompressible ⇒ continuity eq. ⇒ $A\cdot v_{in} = A\cdot v_{out} \Rightarrow v_{out} = v_{in} = \boxed{1\ m/s}$

b) continuity eq.: $A v_{in} = A_r v_r \Rightarrow v_r = \frac{A}{A_r} v_{in} = 4 \cdot 1\ m/s = \boxed{4\ m/s}$

c) Bernoulli's eq. in stand pipe (so $v=0$):

$$\underbrace{\rho g \cancel{h_{bottom}}^{0} + P_x + \tfrac{1}{2}\rho \cancel{v^2}^{0}}_{\text{bottom of standpipe}} = \underbrace{\rho g h_x + \cancel{P_{surface}}^{0 \leftarrow \text{gauge pressure}} + \tfrac{1}{2}\rho \cancel{v^2}^{0}}_{\text{surface of } H_2O \text{ in standpipe}}$$

$$P_x = \rho g h_x = 10^3 \frac{kg}{m^3}\ 10 \frac{m}{s^2}\ 1m = \boxed{10^4\ Pa}$$

gauge pressure

d) Bernoulli's eq.: $\underbrace{\rho g h_y = P_y}_{\text{in stand pipe}}$ ① $\underbrace{\tfrac{1}{2}\rho v_y^2 + P_y = \tfrac{1}{2}\rho v_x^2 + P_x}_{\text{in horizontal pipe}}$ ②

$$② \Rightarrow P_y = \tfrac{1}{2}\rho v_x^2 + P_x - \tfrac{1}{2}\rho v_y^2 = \tfrac{1}{2}\rho(v_x^2 - v_y^2) + P_x$$
$$= \tfrac{1}{2}\, 10^3 \frac{kg}{m^3}\left(1 \frac{m^2}{s^2} - 16 \frac{m^2}{s^2}\right) + 10^4\ Pa$$
$$= -\frac{15}{2}\, 10^3 \frac{kg}{ms^2} + 10^4\ Pa = (-7.5 + 10)\, 10^3\ Pa = 2.5 \times 10^3\ Pa$$
$$① \Rightarrow h_y = P_y / \rho g = \frac{2.5 \times 10^3 \frac{kg}{ms^2}}{10^3 \frac{kg}{m^3}\ 10 \frac{m}{s^2}} = \boxed{0.25\ m}$$

e) Less than, since drag would reduce the pressure in the right side of the horizontal pipe

f) even with viscous drag, water is still incompressible, so by the continuity equation, $\boxed{v_{out} = v_{in}}$

Problem 4 (25 points) Waiting for a wave

A professional surfer is floating on his board waiting for a perfect wave. He is bobbing up and down with a period of 10 seconds due to waves traveling towards the beach at a velocity of 5 m/s.

a) What is the wavelength of the ocean waves passing under the surfer?

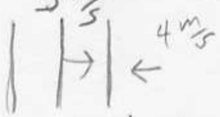
b) A photographer is approaching the surfer from the beach on a water ski that is moving at 4 m/s. At what frequency is the photographer bouncing up and down due to the waves he is passing over?

c) When the photographer reaches the surfer, he realizes that he forgot his camera and turns around to return to the beach at 4 m/s. As he approaches the beach, at what frequency does the photographer bounce up and down?

d) After the photographer returns to the surfer a while later, he realizes he forgot his film, so he speeds back toward the beach at 10 m/s. Now what is the frequency of his bouncing?

a) 

wavelength $= \lambda = vT = 5\frac{m}{s}\, 10s = \boxed{50m}$

b) 

Doppler shift problem

relative v between wave crests and photographer is $5\frac{m}{s} + 4\frac{m}{s} = 9\, m/s$

$$f = \frac{1}{T} = \frac{v}{\lambda} = \frac{9\, m/s}{50m} = \boxed{\frac{9}{50}\, s^{-1}}$$

c) relative v between photographer and waves is $1\, m/s$ since $5\frac{m}{s} - 4\frac{m}{s} = 1\, m/s$

$$f = \frac{v}{\lambda} = \frac{1\, m/s}{50m} = \boxed{\frac{1}{50}\, s^{-1}}$$

d) rel v. $= 5\frac{m}{s} - 10\frac{m}{s} = -5\frac{m}{s}$

$$f = \frac{v}{\lambda} = \frac{5\, m/s}{50m} = \boxed{\frac{1}{10}\, s^{-1}}$$

parts b, c, and d can also be solved using the Doppler shift equation for a moving observer:

$$f_0 = \left(1 \pm \frac{v_0}{v}\right) f$$

f = frequency for stationary observer

f_0 = " " moving observer

v = wave speed relative to beach

v_0 = speed of observer rel. to beach

Problem 5 (20 points) Entropy changes

A 1000kg ice sculpture of the great physicist Lord Kelvin is lost overboard a pleasure boat floating on a large lake (T_{lake} = 27 ºC). The ice is at a temperature of 0ºC. By the time the sculpture is recovered, some of the ice has melted so that only 900kg of ice remains. For this entire problem, assume that the water resulting from the melted ice stays at 0 ºC and does not have sufficient time to warm up to 27 ºC. You may approximate the heat of fusion for water as L_f = 300 kJ/kg

a) During this time, how much heat, $Q_{in\ ice}$, has been transferred from the lake to the ice?
b) What is the change in entropy of the sculpture? Please pay attention to the sign of your answer.
c) How much has the entropy of the lake (not including the sculpture) changed due to the melting ice?
d) What is the net change of the entropy of the whole universe due to this process? Please explicitly state whether this is positive, negative or zero.

a) $Q_{in_{ice}} = L_f \cdot \Delta M = 300 \frac{kJ}{kg}(1000\,kg - 900\,kg) = \boxed{3\times10^4\,kJ}$

b) $\Delta S_{sculp} = \frac{Q_{in\,ice}}{T_{ice}} = \frac{3\times10^4\,kJ}{273\,K} = \boxed{\frac{3\times10^4}{273}\frac{kJ}{K}} \sim 110\frac{kJ}{K}$

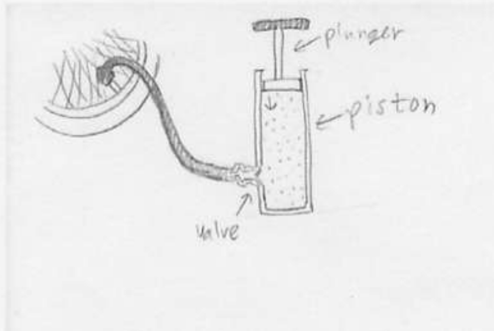
c) $\Delta S_{lake} = \frac{Q_{in\,lake}}{T_{lake}} = \frac{-3\times10^4\,kJ}{300\,K} = \boxed{-10^2\,\frac{kJ}{K}} = -100\frac{kJ}{K}$

d) $\Delta S_{tot.} = \Delta S_{lake} + \Delta S_{ice} = \boxed{-100\frac{kJ}{K} + \frac{3\times10^4}{273}\frac{kJ}{K}} \sim +10\frac{kJ}{K}$

ΔS_{tot} is positive ⇒ entropy in universe increases.

110
273 ⟌ 30000
273
270
273

Problem 6 (45 points) Bicycle pump



A bicyclist is using a bicycle pump to inflate his tire. The pump is designed so that, at the end of each downward stroke, air can escape from the pump through a valve only if the pressure of the gas in the piston reaches or exceeds $P_f = 2$ atm (absolute pressure). The gas starts at $P_i = 1$ atm and $V_i = 0.0001\ m^3$ at the beginning of each downward stroke. Assume that the piston is frictionless and that the air in the piston is an ideal gas.

a) On the first downward stroke, the man pushes the plunger extremely slowly so that the gas in the piston remains in thermal equilibrium with the outside air ($T_{outside} = 27°C$) at all times. What is the piston volume right at the moment when the gas reaches 2 atm?

b) Make a "PV" diagram of the path taken by the air in the piston as P increases from 1 atm to 2 atm; make sure that the end points are exactly right and that the shape of the path is qualitatively correct. Label it "PATH 1."

c) If the amount of work performed on the gas was approximately 6 joules over this path, how much heat Q_{in} was exchanged? Please show your work and check all signs.

d) What is the change in entropy of the gas during this downward stroke from P_i to P_f?

e) On the second downward stroke, the bicyclist pushes the plunger very quickly. Approximately what is Q_{in} during this stroke? Please explain your reasoning.

f) Is the gas temperature at the bottom of the second stroke higher or lower than $T_{outside}$? Explain.

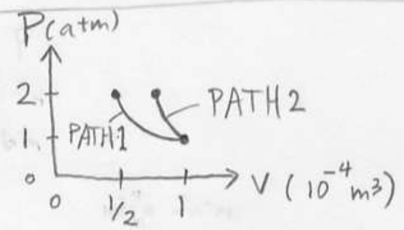
g) On the PV diagram draw the approximate path taken during the second stroke from P_i to P_f. Label it "PATH 2."

h) What is the change in entropy of the gas during this downward stroke?

a) T is constant: ideal gas $\Rightarrow PV = NkT$ T is fixed $\Rightarrow P_iV_i = P_fV_f$

$V_f = \frac{P_i}{P_f} V_i = \frac{1atm}{2atm} 10^{-4} m^3 = \frac{1}{2} 10^{-4} m^3 = \boxed{5 \times 10^{-5} m^3}$

b)
g)



c) ideal gas $\Rightarrow E = (\text{constant}) \cdot NkT$ T is constant $\Rightarrow$ E is constant: $0 = \Delta E = Q_{in} + W_{on}$

so: $\boxed{Q_{in} = -W_{on} = -6\,j}$ (so heat was transfered from the pump to the outside air)

d) $\Delta S = \frac{Q_{in}}{T} = \frac{-6j}{(273+27)K} = \frac{-6j}{300K} = \boxed{-2 \times 10^{-2}\ j/k}$

e) Fast stroke $\Rightarrow$ no time for heat exchange $\Rightarrow \boxed{Q_{in} = 0\,j}$

f) During the first stroke, heat leaves the gas during the compression in order to maintain constant temperature. During the second stroke, the heat can't escape, so the work done on the gas during compression must increase its internal energy. For an ideal gas Energy is proportional to Temp., so $T_f > T_{outside}$ after 2nd stroke

h) no heat is exchanged at any point along PATH 2, so $\Delta S_{a \to b} = \frac{Q_{in\,a \to b}}{T_a} = \boxed{0\ \frac{j}{k}}$