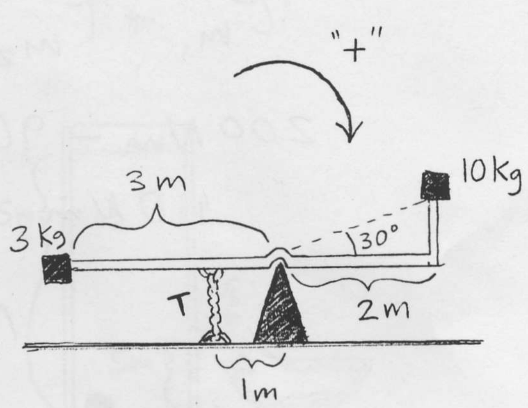
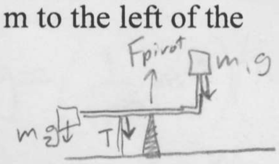
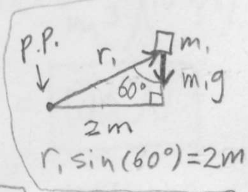
1) Bent see-saw (25 points)



The see-saw shown at the right consists of a triangular base and a bar with a 90° bend 2 m from the pivot point on its right side. A point-like 10 kg mass sits atop the bent arm of the see-saw bar at a position that is at an angle of 30° above the horizontal from the pivot point. A point-like 3 kg weight rests on the left side of the see-saw 3 m from the pivot point. The see-saw is prevented from rotating from its horizontal orientation by a rope tied to a point 1 m to the left of the pivot point; the other end of the rope is tied to a point on the ground directly below the point where it is tied to the see-saw bar. Assume that the see-saw bar itself is massless and not rotating (in other words, it is static).

a) Make a free body diagram of the see-saw bar. Make sure to clearly label all forces.
b) What is the torque on the see-saw bar about the pivot point due to the mass on the right? Use the convention that clockwise torques are positive.
c) What is the torque on the see-saw bar about the pivot point due to the mass on the left? Please check your signs.
d) Find the tension in the rope.
e) What is the largest mass, M_3, that could be added to the bar 1.5 m to the left of the pivot point without causing the bar to rotate counterclockwise?





a)

$F_{m_1} = |\vec{F}_{m_1}| = 10\,kg \cdot g$

$F_{m_2} = |\vec{F}_{m_2}| = 3\,kg \cdot g$

b) $\tau_{m_1} = +(r_1 \sin(60°)) F_{m_1} = (2m)(10\,kg \cdot 10\,\frac{m}{s^2}) = \boxed{+200\ Nm}$

c) $\tau_{m_2} = -r_2 F_{m_2} \cancelto{1}{\sin(90°)} = -(3m)(3\,kg \cdot 10\,\frac{m}{s^2}) = \boxed{-90\ Nm}$

d) N2L rot. $\Rightarrow$ $\tau_{net} = I\alpha$ $\swarrow \alpha = 0$ (static problem)

$\tau_{net} = 0$

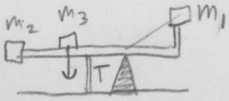
I'll compute all torques about the pivot point of the see-saw

$$\tau_{net} = \tau_{m_1} + \tau_{m_2} + \tau_T + \tau_{pivot} = 0$$

$$+200\ Nm - 90\ Nm + (-1m \cdot T \cancelto{1}{\sin(90°)}) + 0 = 0$$

$$T = \frac{200\ Nm - 90\ Nm}{1m} = \boxed{110\ N}$$

e)



To find the largest M_3, set $T = 0$ so the rope applies no torque.

$\tau_{M_{3\,max}} = -1.5m \cdot M_{3\,max}\, g \cancelto{1}{\sin(90°)}$

$\tau_{net} = 0 \Rightarrow \tau_{m_1} + \tau_{m_2} + \tau_{M_{3\,max}} + \cancelto{0}{\tau_T} + \cancelto{0}{\tau_{pp}} = 0$

(please see back)

$$\tau_{m_1} + \tau_{m_2} + \tau_{m_3} = 0$$

$$200\,Nm - 90\,Nm - (1.5\,m)(M_{3max})(10\,\tfrac{m}{s^2}) = 0$$

$$110\,Nm - 15\,\frac{m^2}{s^2}\,M_{3max} = 0$$

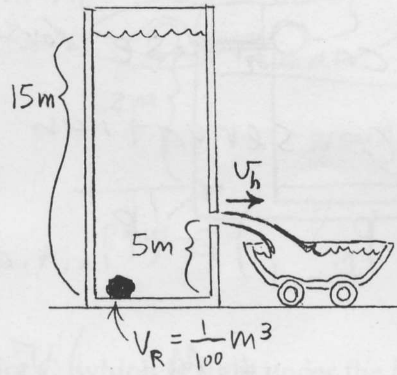
$$M_{3max} = \frac{110\,Nm}{15\,\frac{m^2}{s^2}} = \frac{110\,kg\,\frac{m^2}{s^2}}{15\,\frac{m^2}{s^2}}$$

$$= \frac{110}{15}\,kg$$

$$= \frac{22}{3}\,kg$$

$$= \boxed{7.3\,kg}$$

2) Tank drains into a cart (35 points)

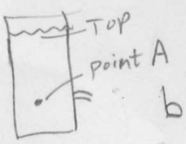


A tank filled to a height of 15 m of water stands next to an empty 20 kg cart as shown in the figure on the right. At time t = 0, a square 0.1 m by 0.1 m hatch on the side of the tank is opened for exactly one second, allowing water to rush out of the hole and fill the cart. All of the dimensions of the tank are much larger than the size of this hatch, so the water level remains essentially unchanged after the hatch is closed again. The hatch is located 5 m from the bottom of the tank. The density of water is $\rho = 1000\ kg/m^3$.

a) There is a rock with volume $V_R = 0.01\ m^3$ sitting at the bottom of the tank. What is the buoyant force on the rock due to the water?
b) What is the pressure in the water inside the tank at the same height as the hole? State whether you are expressing your answer in terms of gauge pressure or absolute pressure and be consistent with your choice for the rest of this problem.
c) What is the speed of the water shooting out horizontally through the hole?
d) How much water (in kg) pours from the hole during the 1 sec that the hatch is open?
e) Assuming that all of the water that leaves the tank lands in the cart and none of it spills, how fast does the filled cart move to the right after the hatch has been closed? Assume the cart's wheels are massless and frictionless.

a) Buoyant force on rock $= V_{rock}\, \rho_{H_2O}\, g = \left(\frac{1}{100} m^3\right)\left(1000 \frac{kg}{m^3}\right)\left(10 \frac{m}{s^2}\right)$

(upward force) $= 100\ kg \frac{m}{s^2} = \boxed{100\ N}$



b) I'll use gauge pressure. so $P_{top} = 0$ (P_{top} is pressure near surface of water)

$$\rho g h_{top} + \tfrac{1}{2}\rho \cancel{v_{top}^2}^{\,0} + P_{top} = \rho g h_A + \tfrac{1}{2}\rho \cancel{v_A^2}^{\,0} + P_A$$

(P_A is pressure in tank at height of hole)

$$\rho g(15m) + 0 + 0 = \rho g(5m) + 0 + P_A$$

$$P_A = \rho g(15m - 5m)$$

$$= \left(1000 \frac{kg}{m^3}\right)\left(10 \frac{m}{s^2}\right)(10m) = \boxed{10^5 \frac{kg}{ms^2}}$$

c) $$\rho g h_{hole} + \tfrac{1}{2}\rho v_h^2 + \cancel{P_{\text{outside hole}}}^{\,0} = \rho g h_{hole} + \tfrac{1}{2}\cancel{\rho v_A^2}^{\,0} + P_A$$

these terms cancel

$$\tfrac{1}{2}\rho v_h^2 = P_A = 10^5 \frac{kg}{ms^2}$$

$$v_h = \sqrt{\frac{2 \cdot 10^5\ kg/ms^2}{1000\ kg/m^3}} = \sqrt{2 \cdot 10^2 \frac{m^2}{s^2}} \sim \boxed{14\ m/s}$$

d) M_{H_2O} = mass of H_2O through hole $= v_h \cdot Area_{hole} \cdot \Delta t\, \rho_{H_2O} = \left(14 \frac{m}{s}\right)(0.1m \times 0.1m)(1s)\left(1000\ kg/m^3\right)$

$= \boxed{140\ kg}$ (see back)

e) inelastic collision: no water spills from cart, so the water and cart "stick together".

I cannot use conservation of K.E. but I can use conservation of momentum

(initial horizontal velocity of the H_2O)

$$(P_{final}) - (P_{initial}) = 0$$

$$(M_{cart} + M_{H_2O})\,V_{final} - (M_{cart}\,\cancelto{0}{V_i} + M_{H_2O}\,V_h) = 0$$

$$V_{final} = \frac{M_{H_2O}\,V_h}{M_c + M_{H_2O}}$$

$$= \frac{140\ \text{kg} \cdot 14\ \tfrac{m}{s}}{20\ \text{kg} + 140\ \text{kg}}$$

$$= \left(\frac{140}{160}\right) 14\ \frac{m}{s}$$

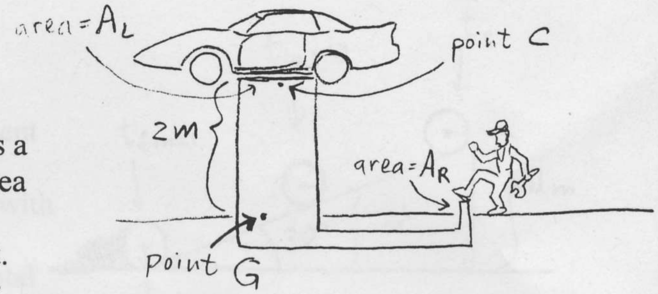
$$= \frac{(14)(14)}{16}\ \frac{m}{s}$$

$$= \frac{7 \cdot 14}{8}\ \frac{m}{s}$$

$$= \frac{98}{8}\ \frac{m}{s}$$

$$\sim \boxed{12\ \frac{m}{s}}$$

3) Mechanical advantage (20 points)



A 1000 Kg car is raised 2 m off of the ground using a hydraulic lift, which consists of a U-shaped tube that has a small area $A_R = 0.001\ m^2$ on the right side and a large area $A_L = 2\ m^2$ on the left side. A car mechanic exerts a downward force on the right side of the lift with his foot. Assume that the hydraulic fluid has the density of water: $\rho = 1000\ kg/m^3$.

a) What is the pressure of the hydraulic fluid located at point C, which is right under the car? State whether you are expressing your answer in gauge pressure or absolute pressure and be consistent with your choice for the rest of this problem.

b) What is the pressure of the fluid at point G, located at ground level on the left side of the lift?

c) What downward force must the mechanic apply to the lift with his foot to keep the car from falling?

a) I will use gauge pressure

right under the car, the pressure is P_c

$$F_{\substack{Net\\ on\ car}} = ma = 0 \quad \text{(car is not accelerating up or down)}$$

$$-F_g + F_{lift} = 0$$

$$F_{lift} = M_c \cdot g = 1000\ kg\ 10\ \frac{m}{s^2} = 10^4\ kg\frac{m}{s^2}$$

$$P_c = F_{lift}/A_L = \frac{10^4\ kg\ \frac{m}{s^2}}{2\ m^2} = \boxed{5,000\ \frac{kg}{m s^2}}$$

b) $$P_G = P_c + \rho_{oil} \cdot g \cdot \Delta h$$

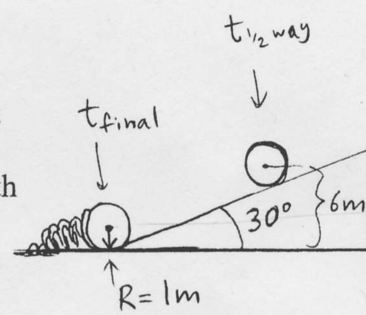
$$= 5,000 \frac{kg}{m s^2} + \left(1000 \frac{kg}{m^3}\right)\left(10 \frac{m}{s^2}\right)(2 m)$$

$$= \boxed{25,000 \frac{kg}{m s^2}}$$

c) oil pressure near mechanics foot is the same as at grond level on other side of lift (so $P_{foot} = P_G$) since the velocity of oil is zero and both locations are at the same height.

$$P_G = P_{foot} = \frac{F_{foot}}{A_R} \Rightarrow F_{foot} = A_R \cdot P_G = (0.001\ m^2)\left(25,000 \frac{kg}{m s^2}\right) = \boxed{25 N}$$

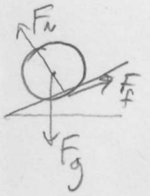
4) Rolling down a ramp (20 points)



A wheel with mass M = 3 Kg, radius R = 1 m and moment of inertia of I = ½ MR^2 about its axis of symmetry rolls down a ramp without slipping. The wheel starts at rest with its center of mass at a height of 11 m above the ground. The ramp is inclined at an angle of 30° from the horizontal and there is a spring at the bottom of the ramp with a rounded piece of metal attached to slow the wheel when it reaches the bottom.

a) When the wheel is halfway down the ramp, so that its center of mass is 6 m above the ground, what is its linear velocity?
b) At this point, what is its angular velocity?
c) At the bottom of the ramp, the spring slows the wheel to a stop right when the wheel reaches the ground, so its center of mass is R = 1 m above the ground. Assuming that the wheel never slides on the ramp, and that the surface of the rounded piece of metal attached to the spring is frictionless, how much energy is stored in the spring at the moment that the wheel comes to rest? Please explain your answer.

a) The wheel rolls without slipping, so I can use conservation of energy since no kinetic or potential energy is lost to dissipative forces.



$$(E_{\frac{1}{2}way}) - (E_o) = 0$$

$$\left(Mgh_{\frac{1}{2}way} + \tfrac{1}{2}Mv_{\frac{1}{2}way}^2 + \tfrac{1}{2}I\omega_{\frac{1}{2}way}^2\right) - \left(Mgh_o + \cancel{\tfrac{1}{2}Mv_o^2}^{\,0} + \cancel{\tfrac{1}{2}I\omega_o^2}^{\,0}\right) = 0$$

$v_{\frac{1}{2}way} = R\cdot\omega_{\frac{1}{2}way}$ ← wheel does not slip

combing these equations:

$$\tfrac{1}{2}Mv_{\frac{1}{2}way}^2 + \tfrac{1}{2}\frac{I v_{\frac{1}{2}way}^2}{R^2} = Mg(h_o - h_{\frac{1}{2}way})$$

$I = \frac{1}{2}MR^2 \Rightarrow$

$$\left(\tfrac{1}{2}M + \tfrac{1}{2}\cdot\tfrac{1}{2}M\frac{R^2}{R^2}\right)v_{\frac{1}{2}way}^2 = Mg(h_o - h_{\frac{1}{2}way})$$

$$\tfrac{3}{4}v_{\frac{1}{2}way}^2 = g(h_o - h_{\frac{1}{2}way})$$

$$v_{\frac{1}{2}way} = \sqrt{\tfrac{4}{3}(10\,\tfrac{m}{s^2})(5m)} = \sqrt{\tfrac{200}{3}}\ \tfrac{m}{s}$$

$$\approx \frac{14}{1.7}\ \tfrac{m}{s} \approx \boxed{8\ \tfrac{m}{s}}$$

b) $$\omega_{\frac{1}{2}way} = \frac{v_{\frac{1}{2}way}}{R} = \frac{8\,m/s}{1m} = \boxed{8\ s^{-1}}$$

c) No energy is lost to heat since the spring is frictionless and the wheel doesn't slip on ramp. So I can use conservation of energy!

$mgh_o = mgh_f + P.E._{spring} \Rightarrow P.E._{spring} = Mg(h_o - h_f) = (3kg)(10\,\tfrac{m}{s^2})(10m) = \boxed{300J}$