Physics 8A (sec 1) FINAL EXAM May 21, 2002

You may use one (1) sheet of paper, not larger than 8.5"x11", as a memory aid, but no other papers and no books. The exam totals 410 points

(20) (1) A block of copper, of mass 2.0 kg, is heated reversibly from 25°C to 100°C. The specific heat of copper (assumed constant) is 386 $J\,(kg)^{-1}K^{-1}$. (a) Calculate the amount Q of heat necessary for this process; (b) Calculate the change ΔS in the entropy of the copper in this process. [(a) = (b) = 10 points each]

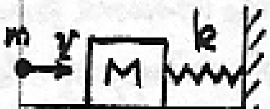
(30) (2) A sample of 2.75 moles of an ideal gas undergoes a reversible isothermal expansion at 77 K, increasing its volume from 1.30 m^3 to 3.40 m^3. Calculate the change ΔS in the entropy of the gas.

(40) (3) A sample of an ideal gas is taken through the following thermodynamic cycle. The initial state A is at pressure $p = 2.5$ kPa, volume $V = 1.0$ m^3, temperature $T = 200$ K. The second state B is at $p = 7.5$ kPa, $V = 3.0$ m^3, temperature T_B. The third state C is at $p = 2.5$ kPa, $V = 3.0$ m^3, temperature T_C. All processes in the cycle are linear on a pV diagram. (a) Draw the pV diagram for this cyclic process; (b) Calculate n, the number of moles of gas present; (c) Calculate temperature T_B; (d) Calculate temperature T_C; (e) Calculate the work W_{AB} done by the gas in step A→B; (f) Calculate the work W_{BC} done by the gas in step B→C; (g) Calculate the work W_{CA} done by the gas in the step C→A; (h) Calculate the total work W done by the gas in the cycle. [Each part = 5 points]

(continued →)

(35) (4) Water flows downward in a vertical pipe whose cross-section area is <u>not</u> constant. At point A of the pipe, the area is 4.0 cm^2 and the speed of the water is 5.0 m/sec. At a lower point B, the area is 8.0 cm^2. (a) Calculate the speed of the water at point B; (b) If point A is 10 meters above point B, calculate the difference in water pressure between points A and B of the pipe. The mass density of water is 1000 kg m^{-3}. (c) Show explicitly that your units in (b) are correct. [a=10, b=15, c=10]

(35) (5) A bullet of mass m and velocity $\underline{v}$ strikes and becomes embedded in a block of mass M. The block is initially at rest

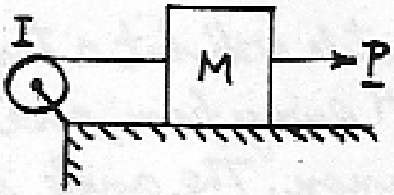


on a horizontal frictionless surface. (a) Calculate the speed of the (block + bullet) immediately after the collision; (b) As shown, the block is attached to a (massless) spring of force constant k. After collision, the block executes simple harmonic motion of amplitude x_m. Calculate the value of x_m. [(a)=10, (b) = 25 points]

(30) (6) A sound wave in air has a wave number of 0.9π m^{-1} and a circular frequency 315π sec^{-1}, with a pressure amplitude of 1.50 Pa. The wave moves in the (+x) direction. (a) Write the equation for this wave giving the pressure variation $\Delta p(x,t)$ at space point x at time t and given the fact that $\Delta p = 0$ at $x=0$ at $t=0$; (b) Calculate the displacement amplitude s_m of the wave; the mass density of air is 1.21 kg m^{-3} [(a) = (b) = 15 points]

(40) (7) A piece of ice of mass 1 kg at a temperature of $-10\,^\circ C$ is placed in a very large lake whose temperature (which may be assumed constant) is $15\,^\circ C$. The ice eventually comes into thermal equilibrium with the liquid water in the lake. The specific heat of ice is $2220\ J\,(kg^{-1})(K^{-1})$, the specific heat of water is $4190\ J\,(kg^{-1})(K^{-1})$, both assumed independent of temperature, and the heat of fusion of ice is $3.33 \times 10^{5}\ J\,(kg^{-1})$. (a) Calculate the change ΔS_{ice} in the entropy of the ice during the entire process described; (b) Calculate the change ΔS_{lake} in the entropy of the lake in the process described; (c) Calculate the change ΔS in the entropy of the system (ice + lake) in the process described; (d) Does the entropy change in part (b) violate the Second Law of Thermodynamics? Justify your answer with an explanation [Each part = 10 points]

(40) (8) A circular wheel of radius R is mounted on a frictionless horizontal axis; the moment of inertia of the wheel is I. A massless inextensible cord is wrapped around the wheel



and is attached to a block of mass M which can slide over a frictionless horizontal surface. A horizontal force $\underline{P}$ is applied to the block. Calculate the magnitude a of the acceleration of the block.

(continued ⟶)

(30)(9) In empty space, a rotating solid spherical star collapses so that its radius decreases by a factor of 10. (a) Calculate the ratio of its final angular frequency ω_f to its initial angular frequency ω_i; (b) Explain why your approach in (a) is valid; (c) Calculate the ratio of the star's final kinetic energy of rotation K_f to its initial kinetic energy of rotation K_i. [Each part = 10 points]

(40)(10) Given a block of mass 1.5 kg located (at rest) at the point $x=0$. A force

$$\underline{F}(x) = (2.5 - x^2)\,\hat{i} \qquad \text{[Newtons]}$$

is applied to the block, which moves along the x-axis to the point $x = 2.0$ meters. (a) Calculate the kinetic energy K of the block at $x = 2.0$ meters; (b) Calculate the maximum value K_{max} of the kinetic energy of the block in the interval $0 \leq x \leq 2.0$ meters. [(a) = 15, (b) = 25 points]

(40)(11) A cannon is fired so its projectile will hit a target which is a horizontal distance H away from, and a vertical distance L above, the cannon. The angle between the initial velocity vector $\underline{v}_0$ of the projectile and the horizontal is 45°. Calculate the magnitude v_0 of the initial velocity required to hit the target.

(continued →)

(30)(12) A satellite of mass m is in a circular orbit around a planet of mass M; the planet exerts a gravitational force on the satellite. (a) Is the total energy of the satellite constant? Justify your answer; (b) Is the linear momentum of the satellite constant? Justify your answer; (c) Is the angular momentum of the satellite constant? Justify your answer. [Each part = 10 points]

SOLUTIONS
PHYSICS 8A FINAL EXAM - MAY 21, 2002

(1)

(1)(a) From the definition of specific heat c, the amount Q of heat required is

$$Q = mc(\Delta T)$$

where ΔT is the increase in temperature. Then

$$Q = (2)(386)(373-298) = 2(386)(75) \text{ Joules}$$

$$\boxed{Q = 5.79 \times 10^4 \text{ J.}}$$

(b) Since the heating process is reversible, the infinitesimal change dS in entropy when heat dQ enters the sample at temperature T is

$$dS = \frac{dQ}{T} \quad \text{where } dQ = mc\,dT$$

Over the finite temperature range (298 K → 373 K), we have

$$\Delta S = \int dS = \int \frac{dQ}{T} = mc\int_{298}^{373} \frac{dT}{T} = mc \ln\left(\frac{373}{298}\right)$$

$$\Delta S = (2)(386)\ln(1.252) = (2)(386)(0.224)$$

$$\boxed{\Delta S = 173 \text{ JK}^{-1}}$$

8A FINAL EXAM SOLUTIONS SP '02 (2)

(2) Since the expansion is isothermal, the temperature T of the gas is constant. Since the gas is ideal, the internal energy E_{int} of the gas depends only on T, so the internal energy of the gas is constant:

$$\Delta E_{int} = 0$$

From the First Law,

$$\Delta E_{int} = Q - W \implies Q = W$$

For an infinitesimal process: $dQ = dW = p\,dV$

For an ideal gas: $pV = nRT \implies p = (nRT)/V$
and

$$dW = nRT\frac{dV}{V}$$

The work W done by the gas during the expansion is

$$W = \int dW = nRT\int \frac{dV}{V} = nRT\left[\ln V\right]_{1.30}^{3.40} = nRT\ln\left(\frac{3.40}{1.30}\right)$$

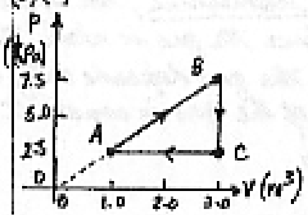
With $n = 2.75$ moles, $T = 77$ K, $R = 8.31$ J (mole)$^{-1}$K^{-1},

$$W = (2.75)(8.31)(77)\ln 2.62 = 1692 \text{ Joules} = Q$$

Since process is isothermal $\Delta S = \int dS = \int \frac{dQ}{T} = \frac{1}{T}\int dQ$

so $$\Delta S = \frac{Q}{T} = \frac{1692 \text{ J.}}{77 \text{ K}} \implies \boxed{\Delta S = 22.0 \text{ JK}^{-1}}$$

(3)(a)



(b) At point A: $pV = nRT$
where $T_A = 200\text{K}$, $p_A = 2.5$ kPa,
$V_A = 1.0\ m^3$, so

$$n = \frac{pV}{RT} = \frac{(2.5\times10^3)(1)}{(8.314)(200)}$$

$$\boxed{n = 1.50 \text{ moles}}$$

(c) $p_B V_B = nRT_B \Rightarrow T_B = \frac{p_B V_B}{nR} = \frac{(7.5\times10^3)(3)}{(1.5)(8.314)} = 1804\text{ K} \cong 1800\text{K}$

(d) $p_C V_C = nRT_C \Rightarrow T_C = \frac{p_C V_C}{nR} = \frac{(2.5\times10^3)(3)}{(1.5)(8.314)} = 601\text{ K} \cong 600\text{K}$

(e) $W_{AB} = \int_A^B p\,dV$ where, from graph, $p = (2500)V$, where $\begin{cases} p \text{ in kPa} \\ V \text{ in } m^3 \end{cases}$

$$W_{AB} = \int_1^3 (2500)V\,dV = 2500\int_1^3 V\,dV = 2500\left[\frac{V^2}{2}\right]_1^3 = (2500)[4] \text{ Joules}$$

$\boxed{W_{AB} = 10{,}000\text{ J.}}$ and $W_{AB} > 0$ because $A \to B$ is an expansion so gas does work on surroundings

(f) $\boxed{W_{BC} = 0}$ since volume is constant in $B \to C$; gas does no work

(g) $W_{CA} = \int_C^A p\,dV = (2500)\int_3^1 dV$ since $p = \text{const.}$ in $C \to A$

$W_{CA} = 2500[1-3] = -5000$ Joules $\Rightarrow \boxed{W_{CA} = -5000\text{ J.}}$

where $W_{CA} < 0$ because gas is compressed, so work is done on gas and thus work done by gas is negative in $C \to A$

(h) $W = W_{AB} + W_{BC} + W_{CA} = (10{,}000 + 0 - 5000) = +5000\text{ J.}$

Total work W done by gas is $\boxed{W = +5000\text{ J.}}$

(4)(a) The equation of continuity relates cross-sectional area A and fluid velocity v at two points:

$$A_1 v_1 = A_2 v_2$$

With $A_1 = 4\text{ cm}^2 = 4\times10^{-4}\ m^2$, $A_2 = 8\text{ cm}^2 = 8\times10^{-4}\ m^2$; $v_1 = 5\text{ m sec}^{-1}$,

$$\boxed{v_2 = 2.5\text{ m/sec}}$$

is the fluid speed at point B where the cross-section area is larger.

(b) Apply Bernoulli's equation to points A and B of the pipe

$$p_A + \tfrac{1}{2}\rho v_A^2 + \rho g h_A = p_B + \tfrac{1}{2}\rho v_B^2 + \rho g h_B$$

where ρ is the mass density of water (1000 kg m^{-3}). Rearranging

$$(p_B - p_A) = \tfrac{1}{2}\rho(v_A^2 - v_B^2) + \rho g(h_A - h_B)$$

$h_A = 10$ m.

$$(p_B - p_A) = \tfrac{1}{2}(1000)[25 - 6.25] + (1000)(9.8)(10)$$

$h_B = 0$ m.

with, from part (a), $v_A = 5\text{ m/sec}$, $v_B = 2.5\text{ m/sec}$, $(h_A - h_B) = 10$ m.

$$(p_B - p_A) = 9375 + 98000 = 107375\text{ Pa}$$

$$\boxed{(p_B - p_A) = 1.07\times10^5\text{ Pa}}$$

(c) To show units are correct in (b), show units of $\frac{1}{2}\rho v^2$ and $\rho g h$ are both Pa; ρ is in kg m^{-3}

$$\tfrac{1}{2}\rho v^2 = (\text{kg m}^{-3})(\text{m}^2\text{ sec}^{-2}) = (\text{kg m sec}^{-2})\text{m}^{-2} = \text{N m}^{-2} = \text{Pa}$$

$$\rho g h = (\text{kg m}^{-3})(\text{m sec}^{-2})(\text{m}) = (\text{kg m sec}^{-2})\text{m}^{-2} = \text{N m}^{-2} = \text{Pa}$$

(5)(a) Linear momentum is conserved in the collision, so

$$mv = (M+m)v'$$

where v' is the speed of (block + bullet) immediately after collision. Then

$$\boxed{v' = \frac{mv}{(M+m)}}$$

(b) After collision, (block + bullet) execute simple harmonic motion with a total energy E equal to kinetic energy of (block + bullet) just after collision, so

$$E = \tfrac{1}{2}(M+m)(v')^2$$

because the block was at rest just before collision. The energy E is related to the amplitude x_m of the simple harmonic motion by

$$E = \tfrac{1}{2}k x_m^2$$

so

$$\tfrac{1}{2}k x_m^2 = \tfrac{1}{2}(M+m)(v')^2 = \tfrac{1}{2}(M+m)\frac{m^2v^2}{(M+m)^2}$$

$$\tfrac{1}{2}k x_m^2 = \tfrac{1}{2}\frac{m^2v^2}{(M+m)}$$

$$x_m^2 = \frac{m^2v^2}{k(M+m)}$$

$$\boxed{x_m = \left[\frac{m^2v^2}{k(M+m)}\right]^{1/2}}$$

M, m in kg
v in m/sec
k in N m^{-1}

is the amplitude of the simple harmonic motion. A units check gives the units for x_m.

(6)(a) The expression for a (longitudinal) sound wave moving in the (+x) direction, expressed in terms of the pressure variation $\Delta p(x,t)$ is given by

$$\Delta p(x,t) = (\Delta p_m)\sin(kx - \omega t)$$

where (Δp_m) is the pressure amplitude, k is the wave number in (meters)$^{-1}$ and ω is the circular frequency in (sec)$^{-1}$. The expression uses a sine (rather than a cosine) because we are given the fact that $\Delta p(0,0) = 0$ at $x = 0$ at $t = 0$. Since $(\Delta p_m) = 1.50$ Pa, $k = 0.9\pi$ m^{-1}, and $\omega = 315\pi$ sec^{-1},

$$\Delta p(x,t) = (1.50)\sin\left[(0.9\pi)x - (315\pi t)\right] \quad [\text{in Pa}]$$

(b) The displacement amplitude s_m is related to the pressure amplitude (Δp_m) by

$$s_m = (\rho v \omega)^{-1}(\Delta p_m)$$

where ρ is the density of the medium (air), $v = (\omega/k)$ is the speed of the wave, so

$$v = (\omega/k) = \frac{315\pi}{0.9\pi} = 350 \text{ m/sec}$$

Then, with $\rho = 1.21$ kg m^{-3}, and using $\Delta p_m = 1.50$ Pa,

$$s_m = \frac{1.50}{(1.21)(350)(315\pi)} = 3.6 \times 10^{-6} \text{ meters}$$

Units check of s_m: $\text{Pa}\,(\text{kg m}^{-3})^{-1}(\text{m sec}^{-1})^{-1}(\text{sec}^{-1})^{-1}$

$(\text{N m}^{-2})(\text{kg})^{-1}\text{m}^{3}\,\text{m}^{-1}\,\text{sec sec} = (\text{kg m sec}^{-2})(\text{kg})^{-1}\text{m}^{-2}\text{m}^{3}\text{m}^{-1}\text{sec}^{2}$

so $\boxed{s_m \text{ is in meters}}$

8A FINAL EXAM SOLUTIONS SPRING 2002 (7)

(7)(a) The ice undergoes three processes: (1) ice warms from 263 K → 273 K; (2) ice melts isothermally at 273 K; (3) water formed by melting ice warms from 273 K → 288 K, at which temperature thermal equilibrium obtains. To calculate ΔS_{ice}:

Process (1): $\Delta S_1 = \int \frac{dQ}{T} = \int_{263}^{273} m_{ice} c_{ice} \frac{dT}{T} = m_{ice} c_{ice} \int_{263}^{273} \frac{dT}{T} = m_{ice} c_{ice} \ln\left(\frac{273}{263}\right)$

$\Delta S_1 = (1)(2220) \ln(1.038) = 82.8 \ JK^{-1}$

Process (2): $\Delta S_2 = \frac{Q}{T} = \frac{m_{ice} L_{f,ice}}{T}$ where $L_{f,ice}$ = heat of fusion of ice

$\Delta S_2 = \frac{(1)(3.33 \times 10^5)}{273} = 1220 \ JK^{-1}$

Process (3): $\Delta S_3 = m_{H_2O} c_{H_2O} \int_{273}^{288} \frac{dT}{T}$ where c_{H_2O} is specific heat of H_2O

$\Delta S_3 = (1)(4190) \ln\left(\frac{288}{273}\right) = (4190)(0.0535) = 224 \ JK^{-1}$

Entropy change $\Delta S_{ice} = \Delta S_1 + \Delta S_2 + \Delta S_3 = 1527 \ JK^{-1} \Rightarrow \boxed{\Delta S_{ice} = 1527 \ JK^{-1}}$

(b) Since lake is large, we assume temperature of lake is constant even though heat Q leaves lake to warm and melt ice. Then

$Q = Q_1(\text{warming ice}) + Q_2(\text{melting ice}) + Q_3(\text{warming water})$

$Q_1 = m_{ice} c_{ice} (\Delta T) = m_{ice} c_{ice} (273 - 263) = (1)(2220)(10) = 22200 \ J.$
$Q_2 = m_{ice} L_{f,ice} = (1)(3.33 \times 10^5) = 3.33 \times 10^5 \ J.$
$Q_3 = m_{H_2O} c_{H_2O} (\Delta T) = (1)(4190)(288 - 273) = (4190)(15) = 62850 \ J.$ (con't →)

8A FINAL EXAM SOLUTIONS SPRING 2002 (7A)

(7) [continued] (b)

$Q = Q_1 + Q_2 + Q_3 = (2.22 \times 10^4) + (3.33 \times 10^5) + (6.825 \times 10^4) \ J.$

$Q = 4.18 \times 10^5 \ J.$ where $Q < 0$ because heat Q leaves lake

With the temperature $T = 288$ K of the lake assumed constant,

$$\Delta S_{lake} = \frac{Q}{T} = \frac{-4.18 \times 10^5}{288} = -1452 \ JK^{-1}$$

$$\boxed{\Delta S_{lake} = -1452 \ JK^{-1}}$$

(c) Total entropy change ΔS for the system (ice + lake) is

$$\Delta S(\text{ice} + \text{lake}) = \Delta S_{ice} + \Delta S_{lake} = (1527 - 1452) \ JK^{-1}$$

$$\boxed{\Delta S(\text{ice} + \text{lake}) = +75 \ JK^{-1}}$$

(d) The fact that, in part (b), the entropy of the lake decreases does not violate the Second Law of Thermodynamics because the Second Law states

$$\Delta S(\text{system} + \text{surroundings}) \geq 0$$

meaning that the entropy change of (system plus surroundings) is never negative. In this example, even though the entropy of the surroundings (the lake) does decrease, the entropy of the (system plus surroundings, that is (ice + lake) does increase (and not decrease). Therefore, the Second Law is not violated.

2A FINAL EXAM SOLUTIONS SPRING 2002 (8)

(8)

$\underline{T}$ = force exerted on rim of wheel by cord

$\tau = RT$ = torque exerted on wheel by cord

$(\underline{\tau} = \underline{R} \times \underline{T} = RT \sin 90° = RT)$

<u>Force diagram of block</u>: $\underline{T}'$ = force exerted on block by cord; $\underline{P}$ = horizontal force applied to block.

$$\underline{T}' \longleftarrow \overset{M}{\bullet} \longrightarrow \underline{P}$$

$\underline{T}'$ = tension in cord, and, since cord is massless and inextensible,

$$|\underline{T}'| = |\underline{T}| = T$$

<u>Newton's 2nd Law for block</u>: $P - T = Ma$, where a = acceleration of block

<u>Newton's 2nd Law for wheel</u>: $\tau = I\alpha$, where $\begin{cases} \alpha = \text{angular acceleration of wheel} \\ I = \text{moment of inertia of wheel} \end{cases}$

Since cord is inextensible, cord does not stretch and slip on wheel, so

$$a = R\alpha \Rightarrow \alpha = (a/R)$$

Then, with $\tau = RT$, $RT = I(a/R) \Rightarrow R^2 T = Ia$

$$T = (Ia/R^2)$$

and

$$P - T = P - \frac{Ia}{R^2} = Ma \Longrightarrow P = Ma + \frac{Ia}{R^2}$$

$$\Rightarrow P = a\left(M + \frac{I}{R^2}\right) \Rightarrow \boxed{a = \frac{P}{\left(M + \frac{I}{R^2}\right)}}$$ is acceleration of block

2A FINAL EXAM SOLUTIONS SPRING 2002 (9)

(9)(a) Since the star is rotating in empty space, there are no external forces and hence no external torques exerted on the star. Therefore the <u>angular momentum</u> L of the star is <u>conserved</u>. With "i" for "initial" and "f" for "final", we have

$$L_i = L_f$$
$$I_i \omega_i = I_f \omega_f$$

where I is the moment of inertia and ω the angular frequency. Since the star is spherical, with M the mass of the star

$$I_i = \tfrac{2}{5} M R_i^2 = 0.4 M R_i^2 \;;\; I_f = \tfrac{2}{5} M R_f^2 = 0.4 M R_f^2$$

where R_i and R_f are the initial and final radii of the star before and after it collapses. We are told that

$$R_f = 0.1 R_i$$

so

$$(0.4MR_i^2)\omega_i = (0.4MR_f^2)\omega_f = (0.4M)(0.01R_i^2)\omega_f$$

and

$$\omega_i(0.4R_i^2) = (0.004R_i^2)\omega_f$$
$$0.4\omega_i = 0.004\omega_f$$
$$\boxed{\omega_f = 100\omega_i} \Rightarrow \boxed{\frac{\omega_f}{\omega_i} = 100}$$

(b) As described above $I_i\omega_i = I_f\omega_f$ because angular momentum is conserved.

(c) Since kinetic energy $K = \frac{1}{2} I \omega^2$, $K_i = \frac{1}{2} I_i \omega_i^2$; $K_f = \frac{1}{2} I_f \omega_f^2$

$$\frac{K_f}{K_i} = \frac{0.5 I_f \omega_f^2}{0.5 I_i \omega_i^2} = \frac{MR_f^2 \omega_f^2}{MR_i^2 \omega_i^2} = \left(\frac{R_f}{R_i}\right)^2 \left(\frac{\omega_f}{\omega_i}\right)^2 = (0.1)^2 (100)^2 = 100$$

$$\boxed{(K_f/K_i) = 100}$$

(10) The work-kinetic energy theorem says that $W = \Delta K$, where W is the work done on the block, and ΔK is the change in the kinetic energy of the block. Since the block is initially at rest (at $x=0$) the block's initial kinetic energy $K_i = 0$, so the final kinetic energy K of the block (at $x = 2.0$ meters) equals the work W. Then

(a) $W = \int \underline{F}(x)\cdot d\underline{x}$ where $\underline{F}(x) = (2.5-x^2)\hat{i}$ and $d\underline{x} = (dx)\hat{i}$

$$W = \int_0^2 (2.5-x^2)\,dx\,(\hat{i}\cdot\hat{i}) = \int_0^2 (2.5-x^2)\,dx = \left[2.5x - \frac{x^3}{3}\right]_0^2$$

$$W = (2.5)(2) - \frac{8}{3} = \frac{15}{3} - \frac{8}{3} = \frac{7}{3} = 2.33 \text{ Joules}$$

Hence kinetic energy $\boxed{K \text{ of block is } 2.33 \text{ Joules}}$ at $x = 2.0$ meters

(b) To find K_{max}, we need $K(x)$, kinetic energy at any point x between $x=0$ and $x=2$. If $W(x)$ is work done from $x=0 \to x$,

$$K(x) = W(x) = \int_0^x (2.5-x^2)\,dx = \left[2.5x - \frac{x^3}{3}\right]_0^x = \left(2.5x - \frac{x^3}{3}\right)$$

To find K_{max}, set

$$\frac{dK(x)}{dx} = 0 \Rightarrow \frac{d}{dx}\left[2.5x - \frac{x^3}{3}\right] = (2.5 - x^2) = 0 \Rightarrow x = (2.5)^{1/2}$$

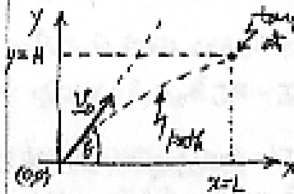
(where the negative square root is ignored because $0 \le x \le 2$). Then

$$K_{max} = K(x = 2.5) = \left[(2.5)(2.5)^{1/2} - \frac{(2.5)^{3/2}}{3}\right]$$

$$K_{max} = \left[(2.5)^{3/2} - \frac{1}{3}(2.5)^{3/2}\right] = \left(\frac{2}{3}\right)(2.5)^{3/2} = \left(\frac{2}{3}\right)(3.95)$$

$$\boxed{K_{max} = 2.64 \text{ Joules}}$$

(11) This is a projectile problem, shown in the drawing.



We have x and y as fns of time:

$$\begin{cases} x(t) = x_0 + (v_0\cos\theta)t \\ y(t) = y_0 + (v_0\sin\theta)t - \frac{1}{2}gt^2 \end{cases}$$

where initial velocity v_0 makes angle θ with horizontal.

Put the origin ($x=0, y=0$) at location of cannon, so, with $\theta = 45°$

$$\begin{cases} x(t) = (v_0\cos\theta)t \\ y(t) = (v_0\sin\theta)t - \frac{1}{2}gt^2 \end{cases}$$

$$\begin{cases} x(t) = (0.707\,v_0)t \\ y(t) = (0.707\,v_0)t - \frac{1}{2}gt^2 \end{cases}$$

These are two equations in the variables v_0 and t. For the projectile to pass through the point ($x=L$, $y=H$), we have

$$\begin{cases} L = (0.707\,V_0)t \\ H = (0.707\,V_0)t - \frac{1}{2}gt^2 \end{cases} \Longrightarrow \boxed{t = \frac{L}{(0.707\,V_0)}}$$ is time for projectile to reach target

Then equation for v_0 becomes:

$$H = (0.707\,v_0)\left(\frac{L}{(0.707\,v_0)}\right) - \frac{1}{2}g\left(\frac{L}{0.707\,v_0}\right)^2 = L - \frac{1}{2}g\frac{L^2}{(0.5\,v_0^2)}$$

$$\frac{gL^2}{v_0^2} = L - H \Longrightarrow v_0^2 = \frac{gL^2}{(L-H)} \Longrightarrow \boxed{v_0 = \left[\frac{gL^2}{(L-H)}\right]^{1/2}}$$

Note: If $L \to H$, $v_0 \to \infty$. This means that, with cannon at 45° to horizontal, projectile with non-infinite initial velocity cannot reach target because path is "pulled down" by gravity.

(12) (a) The total energy E of the satellite is the sum of its kinetic energy and gravitational potential energy:

$$E = K + U = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

where r is the radius of the orbit and v the linear velocity. The only force acting on the satellite is gravity, so there is no other force doing work on the satellite. Since r = const. for circular orbit and v = const., E = constant.

(b) Newton's Second Law: $\underline{F} = (d\underline{p}/dt)$ where $\underline{p}$ is linear momentum. There is a force (gravity) acting on the satellite, so $|\underline{F}| \neq 0$, meaning that the linear momentum vector $\underline{p}$ is not constant. In this circular orbit, $\underline{p}$ changes direction continuously while the magnitude $|\underline{p}|$ is unchanged.

(c) The gravitational force $\underline{F}$ acting on the satellite acts along the line connecting planet and satellite. The torque $\underline{\tau}$ due to force $\underline{F}$ is

$$\underline{\tau} = \underline{r} \times \underline{F}$$

But since $\underline{F}$ is antiparallel to position vector $\underline{r}$, torque

$$\underline{\tau} = 0.$$

Also, rate of change of angular momentum $\underline{L}$ of satellite is

$$\frac{d\underline{L}}{dt} = \underline{\tau} = 0$$

so angular momentum $\underline{L}$ of satellite is constant in time.