Physics 8A (Sec 1) Midterm Exam #1 March 4, 2002

You may use one (1) card, not larger than 3" × 5", as a memory aid, but no other papers, and no books. Exam totals ~~160~~ 150 points

(20) (1) An elevator of mass m is supported by a cable of negligible mass. The elevator is pulled upward with acceleration a. (a) Calculate the force F exerted on the elevator by the cable; (b) Suppose that the same elevator is moving downward with acceleration of magnitude a. Calculate the force F' exerted on the elevator by the cable; (c) In part (a), calculate F if the mass of the elevator is 2000 kg and its upward acceleration is 0.1 m sec^{-2}. [Part (a) = 8 pts, (b) = 8 pts, (c) = 4 pts]

(20) (2) An automobile of mass m (moving on a horizontal road) applies its brakes (locking its tires) when moving with speed v. The coefficient of friction between tires and road is μ. (a) Calculate the distance d the auto travels before stopping; (b) Calculate the value of d for an auto moving at 15 m/sec on a road for which $\mu = 0.8$; (c) Show explicitly that your units in part (a) are correct. [(a) = 15 pts, (b) = 3 pts, (c) = 2 pts.]

(20) (3) A force $\underline{F}(x) = (2x)\hat{i} + 3\hat{j}$ moves a block along a frictionless horizontal table top from $x = 1$ m. to $x = 3$ m. Here $|\underline{F}|$ is in Newtons, x is in meters horizontally and y is in meters vertically. (a) Calculate the work done by force $\underline{F}$; (b) As the block moves along the x-axis, calculate the work done on the block by the normal force exerted on the block by the table; (c) As the block moves, starting from rest at $x = 1$ meter, its kinetic energy K increases. Calculate the kinetic energy of the block at the point $x = 3$ meters. [(a) = 10, (b) = 5, (c) = 5 points]

(continued ⟶)

(20)(4) A man on a bicycle moves with constant speed 9.0 m/sec in a circle of radius 25.0 meters; the mass of (man plus bicycle) is 85.0 kg. (a) Describe in words the force $\underline{F}$, exerted on the bicycle by the road, that keeps the bicycle moving in a circle; (b) Give the direction of $\underline{F}$, explaining your reasoning; (c) Calculate the magnitude of force $\underline{F}$ in part (a); (d) Calculate the magnitude of the total force exerted on the bicycle by the road. [Part (a) = 4, (b) = 4, (c) = 6, (d) = 6 points]

(20)(5) A crate of mass 100 kg is to be pushed with constant speed up a frictionless incline which makes an angle of 30° with the horizontal. (a) Calculate the magnitude of the horizontal force $\underline{F}$ required; (b) Calculate the magnitude of the force $\underline{F}'$ exerted by the crate on the ramp. [(a) = 10, (b) = 10 points]

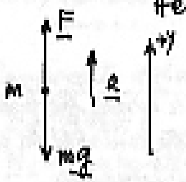
(20)(6) A boat of mass 2000 kg is moving at a speed of 25 m/sec (about 55 mph) when the engine is shut off. The magnitude f of the frictional force between the boat and the water at time t is given by $f = 70t$, where f is in Newtons and is directed opposite to the direction of the velocity $\underline{v}$ of the boat. Calculate the time necessary for the boat to slow down to a speed of 12.5 m/sec.

(continued ⟶)

(30)(7) A cannon ball is fired from a point which is a horizontal distance d from the foot of a vertical cliff of height h. The initial velocity of the cannon ball is $\underline{v}_0$ and the angle $\underline{v}_0$ makes with the horizontal is θ_0. It is desired that the cannon ball be at the highest vertical point of its trajectory when it lands at the top (at the edge) of the cliff. Calculate the values of $|\underline{v}_0|$ and θ_0 which will achieve the desired result. (You may leave your answer in terms of two clearly-identified equations which can be solved simultaneously for v_0 and θ_0.)

Physics 8A (Sec. 1) Solutions to Midterm #1 March 4, 2002

(1)(a) Draw the force diagram for the elevator, as shown:



Here $\underline{F}$ is the upward force exerted on elevator by cable

$m\underline{g}$ is the gravitational force on elevator

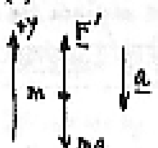
$\underline{a}$ = acceleration upward of elevator

Net force on elevator = $\underline{F} + m\underline{g} = F\hat{j} - mg\hat{j} = ma\hat{j}$

Then, solving for $|\underline{F}| = F$, $\Rightarrow$ $F - mg = ma$

$$\boxed{F = m(g+a)}$$

(b) Elevator moves downward, so $\underline{a} = a(-\hat{j})$, and $\underline{F}'$ is force exerted on elevator by cable, where



$\underline{F}' + m\underline{g} = m\underline{a}$

$F'\hat{j} - mg\hat{j} = ma(-\hat{j})$

$F' - mg = -ma$

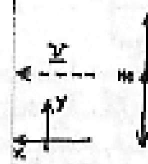
$$\boxed{F' = m(g-a)}$$

(c) In part (a), $m = 2000$ kg, $a = 0.1$ m sec^{-2}, $g = 9.8$ m sec^{-2}

$F = |\underline{F}| = m(g+a) = (2000)(9.9)$

$$\boxed{F = 19800 \text{ Newtons}}$$

(2) For the sake of concreteness, assume the car is moving to the left horizontally. Force diagram of car of mass m:



$\underline{N}$ = normal force exerted on tires by road

$\underline{f}$ = frictional force exerted on tires by road

$m\underline{g}$ = gravitational force exerted on tires (car)

Since there is no motion in y-direction, $a_y = 0$, so

$N - mg = ma_y = 0 \Longrightarrow \boxed{N = mg}$

Frictional force $f = |\underline{f}| = \mu N = \mu mg$. Then Newton's 2nd Law says

$\underline{f} = m\underline{a} \Rightarrow f(-\hat{x}) = m\underline{a} \Longrightarrow \mu mg(-\hat{x}) = m\underline{a}$

and $\underline{a} = \mu g(-\hat{x}) \Longrightarrow a\hat{x} = \mu g(-\hat{x}) \Longrightarrow \boxed{a = -\mu g}$

where $a < 0$ means car is decelerating. We know the initial velocity v_0, the final velocity v_f, so, with $v_0 = v$, $v_f = 0$,

$v_f^2 = v_0^2 + 2ad \Longrightarrow 0 = v^2 - 2\mu g d$

$\boxed{d = (v^2/2\mu g)}$ is the desired answer

(b) If $v = 15$ m/sec, $\mu = 0.8$, $g = 9.8$ m/sec^2, $\boxed{d = 14.3 \text{ meters}}$

(c) Units of d are units of d^2 ($m^2 sec^{-2}$) divided by units of g ($m\ sec^{-2}$) since μ is dimensionless, so d is in meters, as it should be because d is a length.

8A MT#1 SPRING 2002 SOLNS (3)

(3)(a) The force $\underline{F}(x) = (2x)\hat{i} + 3\hat{j}$ and the motion is along the x-axis from $x=1$ to $x=3$, so the work done by $\underline{F}$ is

$$W = \int \underline{F}(x)\cdot d\underline{x} = \int_1^3 \left[2x\hat{i} + 3\hat{j}\right]\cdot (dx)\hat{i}$$

since the infinitesimal displacement $d\underline{x} = (dx)\hat{i}$. Then

$$W = \int_1^3 \left[2x\,dx\,(\hat{i}\cdot\hat{i}) + 3\,dx\,(\hat{i}\cdot\hat{j})\right] \quad \text{where} \begin{cases}(\hat{i}\cdot\hat{i}) = 1\\ (\hat{i}\cdot\hat{j}) = 0\end{cases}$$

so $$W = \int_1^3 2x\,dx = 2\left[\frac{x^2}{2}\right]_1^3 = 2\left(\frac{9}{2} - \frac{1}{2}\right) = 8 \text{ Joules}$$

(b) The normal force $\underline{N}$ exerted on the block by the table is $\underline{N} = N\hat{j}$ because $\underline{N}$ is vertical and so in the y-direction. The work W' done by force $\underline{N}$ is

$$W' = \int \underline{N}\cdot d\underline{x} = \int (N\hat{j})\cdot(dx\,\hat{i}) = \int N\,dx\,(\hat{j}\cdot\hat{i}) = 0$$

because $(\hat{j}\cdot\hat{i}) = 0$ and so each element of work $(N\,dx)(\hat{j}\cdot\hat{i}) = 0$.

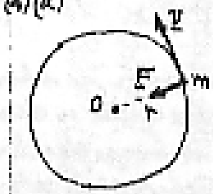
(c) From the Work-Kinetic Energy Theorem, the change ΔK in kinetic energy equals the work W done, so

$$W = K_{final} - K_{initial} = K_{final} \quad \text{since } K_{initial} = 0$$

because the block starts (at $x = 1$ m.) at rest. Then K_{final} (at $x = 3$ m.) equals $W = 8$ Joules, so $\boxed{K_{final} = 8\text{ J.}}$

8A MT#1 SPRING 2002 SOLUTIONS (4)

(4)(a)



The force F is the frictional force exerted on the bicycle tires by the road because the road is the only entity which is in contact with the bicycle.

(b) Since the (man + bicycle) move in a circle the force $\underline{F}$ must be radially inward toward the center O of the circle. Thus $\underline{F}$ is the centripetal force in this situation

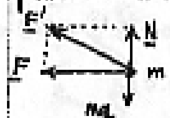
(c) The magnitude of the centripetal force F is given by

$$F = (mv^2/r) \quad (1)$$

(But F is due to friction; do we need to know the coefficient of friction? No, because the centripetal force is given by (1) above).

$$F = \frac{(85)(9)^2}{25} = 275 \text{ Newtons} \implies \boxed{F = 275\text{ N.}}$$

(d) The total force $\underline{F}'$ exerted on the (man + bicycle) is the vector sum of the centripetal force $\underline{F}$ (exerted on bicycle by road) plus the normal force $\underline{N}$ (exerted on bicycle by road). If the plane of the circular path is normal to the paper, situation is as



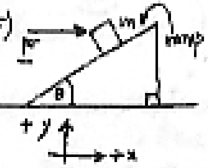
shown at left. The weight $|m\underline{g}|$ (which is not exerted on bicycle by road) equals $|\underline{N}|$, so $N = mg$. Then force $\underline{F}' = (\underline{F} + \underline{N})$ so

$mg = (85)(9.8)$

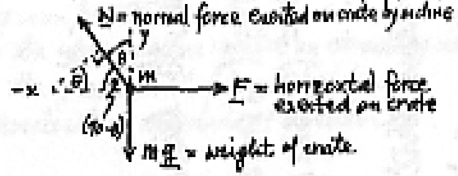
$mg = 833$ N.

$$|\underline{F}'| = (F^2 + N^2)^{1/2} = \left[(275)^2 + (833)^2\right]^{1/2} = (769514)^{1/2}$$

$$\boxed{|\underline{F}'| = 877 \text{ Newtons}}$$

(5) 

(a) With the axes shown, we draw the force diagram of the crate



<u>Components of $\underline{N}$ are:</u>

$\underline{N}_y = (N\cos\theta)\hat{y}$

$\underline{N}_x = (N\cos(90-\theta))(-\hat{x}) = (N\sin\theta)(-\hat{x})$

<u>Apply Newton's Second Law to crate of mass m:</u>

<u>Horizontal</u> (x-axis): $F - N\sin\theta = ma_x = 0$

<u>Vertical</u> (y-axis): $N\cos\theta - mg = ma_y = 0$

since speed of crate is <u>constant</u>, so $a_x = 0$, $a_y = 0$

These equations become $\begin{cases} F = N\sin\theta & (1) \\ mg = N\cos\theta & (2) \end{cases}$

<u>Dividing</u> (1) by (2) gives $(F/mg) = (\sin\theta)/(\cos\theta) = \tan\theta \Rightarrow \boxed{F = mg\tan\theta}$

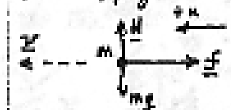
If $m = 100$ kg, $g = 9.8$ m/sec², $\theta = 30°$, $\Rightarrow \boxed{F = 566 \text{ Newtons}}$

(b) <u>From Newton's Third Law</u>, the magnitude F' of the force exerted on the ramp by the crate equals the magnitude $|\underline{N}|$ of the force exerted on the crate by the ramp:

$$F' = N = \frac{mg}{\cos\theta} = \frac{980}{0.866}\text{ N.} = 1132\text{ N.} \Rightarrow \boxed{F' = 1132\text{ N.}}$$

(Note that the weight of the crate is the gravitational force exerted on the crate <u>by the Earth</u>, so the weight is <u>not</u> a force exerted on the ramp.)

(6) We apply Newton's Second Law to the boat of mass m.



Ignoring the upward normal force $\underline{N}$ exerted by the water and the weight $m\underline{g}$ of the boat, the only horizontal force exerted on the boat after the engine is shut off is the frictional force $\underline{f}$. If we say v is the speed of the boat at time t seconds after the engine stops, Newton's 2nd Law becomes

$$-f = ma = m\frac{dv}{dt} \Rightarrow m\,dv = -f\,dt = -\alpha t\,dt \qquad (1)$$

where here α has the numerical value 70 N sec⁻¹. If v_0 is the speed of the boat at time $t = 0$ (the instant the engine stops) and $t = t_f$ is the time at which the boat has the speed $v_f = (v_0/2)$ because the speed initially was 25 m/sec and the final speed is 12.5 m/sec, we want to find t_f.

Integrating equation (1),

$$\int_{v_0}^{v_f} dv = (-m/\alpha)^{-1}\int_{t=0}^{t_f} t\,dt = (-m/\alpha)^{-1}\int_0^{t_f} t\,dt = (-m/\alpha)^{-1}\frac{t_f^2}{2}$$

so $(v_f - v_0) = (-m/\alpha)^{-1}(\tfrac{1}{2})t_f^2 \Rightarrow \boxed{t_f^2 = 2(v_f - v_0)(-\tfrac{m}{\alpha})}$ where $v_0 > v_f$

Since $v_f = 12.5$ m/sec, $v_0 = 25.0$ m/sec, $m = 2000$ kg, $\alpha = 70$ N sec⁻¹,

$t_f^2 = 2(12.5 - 25.0)(-\frac{2000}{70}) = (714)$ sec² ; $t_f = (714)^{1/2}$ sec.

$\boxed{t_f = 26.7 \text{ sec.}}$ is the time required for the boat to slow from 25.0 m/sec to 12.5 m/sec.

8A MT #1 SPRING 2002 SOLUTIONS

(7)

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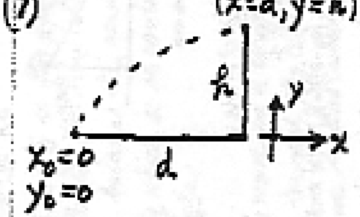

For a projectile launched with initial velocity $\underline{v}_0$ at an angle θ_0, the x- and y-coordinates as a function of time are

$$\begin{cases} x(t) = x_0 + (v_0 \cos\theta_0)t \\ y(t) = y_0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \end{cases}$$

We take the launch point (x_0, y_0) as the origin, so $x_0=0$, $y_0=0$, so

$$\left.\begin{cases} x(t) = (v_0 \cos\theta_0)t \\ y(t) = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \end{cases}\right\} \text{where the trajectory passes through the point } (x=d,\ y=h)$$

This means that it must be true that, at a particular time t,

$$\begin{cases} d = (v_0 \cos\theta_0)t \\ h = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \end{cases}$$

We are told that the cannon ball is at the point (d, h) when the projectile is at the top of its trajectory, at which point, its y-component v_y of velocity is zero. Therefore, when $y=h$,

$$v_y = (dy/dt) = (v_0 \sin\theta_0) - gt = 0 \implies t = (1/g)(v_0 \sin\theta_0)$$

as the time at which the projectile is at the top of its path, that is, the time at which the projectile is at the point (d, h).

Thus
$$\begin{cases} d = (v_0 \cos\theta_0)(1/g)(v_0 \sin\theta_0) \\ h = (v_0 \sin\theta_0)(1/g)(v_0 \sin\theta_0) - (g/2)(1/g)^2(v_0 \sin\theta_0)^2 \end{cases}$$

Simplifying,

$$\left.\begin{cases} d = (1/g)\, v_0^2 \sin\theta_0 \cos\theta_0 \\ h = (1/2g)\, v_0^2 \sin^2\theta_0 \end{cases}\right\} \Rightarrow \begin{cases} gd = v_0^2 \sin\theta_0 \cos\theta_0 \\ 2gh = v_0^2 \sin^2\theta_0 \end{cases}$$

as two equations in the required two unknowns v_0 and θ_0.