Ö. Savaş NAME: **SOLUTIONS** 9:10-10:00, Friday, October 31, 2008

ME 106 FLUID MECHANICS

EXAM 2 – open book, open notes, no external communication

1.(30%)

Consider capillary waves on the surface of a liquid in the absence of gravity. On the assumption that the wave speed c can only depend on the surface tension σ, liquid density ρ and the wavelength λ; using dimensional analysis, deduce dependence of c on the wavelength λ.

$$[c] = \frac{L}{T} \qquad [\sigma] = \frac{ML}{T^2}\frac{1}{L} \qquad [\rho] = \frac{M}{L^3} \qquad [\lambda] = L \Longrightarrow c \sim \sqrt{\frac{\sigma}{\lambda\rho}}$$

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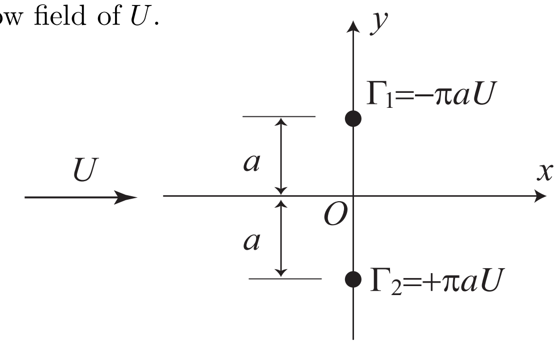
grading table

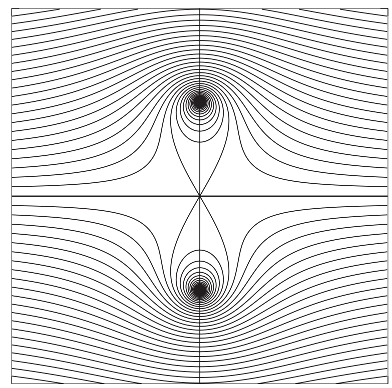
1 (30)	**2** (35)	**3** (35)	**total** (100)

2.(5+5+10+5+5+5=35%)

Consider a line vortex of strength $\Gamma_1 = -\pi a U$ located at $(x, y) = (0, +a)$ and another of strength $\Gamma_2 = +\pi a U$ located at $(x, y) = (0, -a)$ in an otherwise uniform flow field of U.



- **(a)** Construct the potential function of the flow field.
- **(b)** Construct the stream function of the flow field.
- **(c)** Determine velocity on the plane $y = 0$.
- **(d)** Determine the pressure coefficient on the plane $y = 0$.
- **(e)** Determine the stagnation point(s) of the flow.
- **(f)** Sketch the streamline pattern.



Total potential function construction

$$\phi = \phi_U + \phi_{\Gamma_1} + \phi_{\Gamma_2} = Ux + \frac{\Gamma_1}{2\pi} tan^{-1}\left(\frac{y-a}{x}\right) + \frac{\Gamma_2}{2\pi} tan^{-1}\left(\frac{y+a}{x}\right) = \frac{aU}{2}\left[\frac{2x}{a} - tan^{-1}\left(\frac{y-a}{x}\right) + tan^{-1}\left(\frac{y+a}{x}\right)\right] \quad (1)$$

Total stream function construction

$$\psi = \psi_U + \psi_{\Gamma_1} + \psi_{\Gamma_2} = Uy - \frac{\Gamma_1}{2\pi} ln\sqrt{x^2+(y-a)^2} - \frac{\Gamma_2}{2\pi} ln\sqrt{x^2+(y+a)^2} = \frac{aU}{4}\left[\frac{4y}{a} + ln\frac{x^2+(y-a)^2}{x^2+(y+a)^2}\right] \quad (2)$$

Velocity field $\mathbf{u} = (u, v) = (\phi_x, \phi_y) = (\psi_y, -\psi_x)$

$$\left(\frac{u}{U}, \frac{v}{U}\right) = \left[1 + \frac{a}{2}\left(\frac{y-a}{x^2+(y-a)^2} - \frac{y+a}{x^2+(y+a)^2}\right),\ -\frac{a}{2}\left(\frac{x}{x^2+(y-a)^2} - \frac{x}{x^2+(y+a)^2}\right)\right] \quad (3)$$

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At $y = 0$

$$\left(\frac{u}{U}, \frac{v}{U}\right) = \left[1 - \frac{a^2}{x^2+a^2}, 0\right] \qquad C_p = 1 - \frac{u^2}{U^2} = \frac{2x^2+1}{(x^2+1)^2} \qquad C_p(x=0) = 1,\ \text{stagnation point}$$

Closed streamline through the stagnation point

$$\psi = \frac{aU}{4} ln\left[e^{4y/a}\frac{x^2+(y-a)^2}{x^2+(y+a)^2}\right] = 0 \quad (4)$$

The closed streamline is a pair of tear-drops touching at the origin, with y-axis intercepts at $\pm 1.2a$.

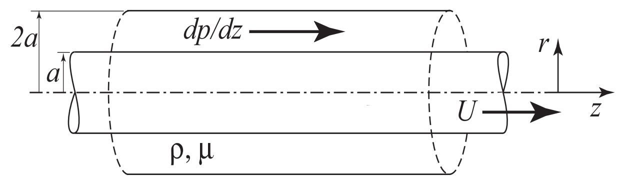
3.(10+15+10=35%)

Consider a rod of radius a moving inside a fixed cylinder of radius $2a$. The gap is filled with a fluid of density ρ and viscosity μ. The rod velocity is U. The axial pressure gradient is dp/dz. The rod and the cylinder are concentric.

(a) Obtain the velocity profile in the annular gap.

(b) Determine the relation between the U and dp/dz when the shear stress on the cylinder vanishes.

(c) Determine the shear stress on the rod when the shear stress on the cylinder vanishes.



Hint: For convenience, define $P = \frac{1}{4\mu}\frac{dp}{dz}$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{1}{\mu}\frac{dp}{dz} \Longrightarrow u = \frac{1}{4\mu}\frac{dp}{dz}r^2 + A\ln r + B = Pr^2 + A\ln r + B$$

$$u = P(r^2 - a^2) - \frac{U + 3a^2P}{\ln 2}\ln(r/a) + U$$

In dimensionless form

$$\frac{u}{U} = 1 + \frac{a^2P}{U}\left(\frac{r^2}{a^2} - 1\right) - \frac{1 + 3a^2P/U}{\ln 2}\ln(r/a)$$

$$\frac{du}{dr} = 2Pr - \frac{U + 3a^2P}{\ln 2}\frac{1}{r}$$

On the cylinder, $r = 2a$

$$\left.\frac{du}{dr}\right|_{r=2a} = 4Pa - \frac{U + 3a^2P}{\ln 2}\frac{1}{2a} = 0 \Longrightarrow P = \frac{U}{(8\ln 2 - 3)a^2} = 0.393\,\frac{U}{a^2}$$

On the rod, $r = a$

$$\frac{\tau}{\mu} = \left.\frac{du}{dr}\right|_{r=a} = 2Pa - \frac{U + 3a^2P}{\ln 2}\frac{1}{a} = \frac{-6U}{(8\ln 2 - 3)a}$$