Physics 7C Section 1
Fall Semester 2002
Second Midterm

November 4, 2001 (6:15-7:45 pm)

Instructions

1. This is a **closed book** exam. You are allowed to bring along only pens, pencils, scientific calculator, and blue books.
2. **Write your name, Discussion Section # and SID#** on the top of all materials you intend to hand in and want to be graded.
3. Read all questions carefully before attempting them. Questions do not carry equal points. Try the questions you find easier first. Partially credits will be given for equations only if you can indicate how they can be used to solve the problem.
4. While cleanliness and legibility of your hand-writing will not get your extra credit, they will help to make sure that your answers get the credit they deserve. In case you make mistakes be sure to cross them out so they will not be mistaken as your answer. It helps to underline your final answers. Always give your answers in the proper units.

You are provided with the following constants for your reference. There is no guarantee that they will all be required in solving the problems.

Fundamental Constants

Quantity	Symbol	Approximate Value
Speed of light in vacuum	c	3.00×10^8 m/s
Gravitational constant	G	6.67×10^{-11} N·m²/kg²
Avogadro's number	N_A	6.02×10^{23} mol^{-1}
Gas constant	R	8.315 J/mol·K = 1.99 cal/mol·K = 0.082 atm·liter/mol·K
Boltzmann's constant	k	1.38×10^{-23} J/K
Charge on electron	e	1.60×10^{-19} C
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m²·K⁴
Permittivity of free space	$\epsilon_0 = (1/c^2\mu_0)$	8.85×10^{-12} C²/N·m²
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A
Planck's constant	h	6.63×10^{-34} J·s
Electron rest mass	m_e	9.11×10^{-31} kg = 0.000549 u = 0.511 MeV/c^2
Proton rest mass	m_p	1.6726×10^{-27} kg = 1.00728 u = 938.3 MeV/c^2
Neutron rest mass	m_n	1.6749×10^{-27} kg = 1.008665 u = 939.6 MeV/c^2
Atomic mass unit (1 u)		1.6605×10^{-27} kg = 931.5 MeV/c^2

The binding energy of the electron in the n=1 level in the hydrogen atom=13.6eV.

You are given the following equations for your reference. There is no guarantee that you will need them in solving the problems.

1. Intensity of **Fraunhofer diffraction** of light of wavelength λ by a **single slit** of width a: $I=I_o[\sin\alpha/\alpha]^2$ where $\alpha=\pi a\sin\theta/\lambda$.
2. Intensity of **Fraunhofer diffraction** of light of wavelength λ by **two slits** of same width a separated by distance d: $I=I_o[\sin\alpha/\alpha]^2\cos^2\delta$ where $\alpha=\pi a\sin\theta/\lambda$ and $\delta=\pi d\sin\theta/\lambda$
3. Intensity of **Fraunhofer diffraction** of light of wavelength λ by a **grating** of N lines of same width separated by distance d: $I=I_o[\sin N\beta/\sin\beta]^2$ where $\beta=\pi d\sin\theta/\lambda$
4. **Resolving power** of a diffraction grating of N lines$=\lambda/\Delta\lambda=Nm$ where m=order of diffraction.
5. **Rayleigh Criterion**: two images are just resolvable by a circular lens of diameter D when they are separated by the angle $\theta=1.22\lambda/D$.
6. **Brewster angle** θ_p between two media of refractive indexes n_2 and n_1: $\tan\theta_p=n_2/n_1$.
7. **Planck's Radiation Law** for a Black-Body Radiator:

$$I(\lambda,T)=\frac{2\pi hc^2\lambda^{-5}}{e^{hc/\lambda kT}-1}$$

8. **Wien's Displacement Law**: $\lambda_p T=2.90\times10^{-3}$ m.K where T is the temperature and λ_p =the wavelength where the intensity of emission is maximum.
9. Planck's quantum hypothesis: $E=hf$.
10. **De Broglie's** wave-matter duality hypothesis: $p=h/\lambda$.
11. **Rydberg** series for emission from the hydrogen atom:

$$\frac{1}{\lambda}=R\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$$ where $R=1.0974\times10^7\ m^{-1}$ and $n>m\geq1$ are both integers.

12. **Bohr's model** of Hydrogen atom: electron can be stable in stationary orbits with angular momentum quantized into: $L=mvr=nh/2\pi$.
13. Energy levels of electron in Bohr's model of hydrogen atom:

$$E_n=-\left(\frac{me^4}{8\varepsilon_o^2h^2}\right)\left(\frac{1}{n^2}\right)$$

14. **Bohr radius** of the electron in the n=1 energy level : $a_B=\frac{h^2\varepsilon_o}{\pi me^2}=0.529\times10^{-10}$ m.
15. The Kinetic Energy K of the photoelectrons ejected by photons of frequency (f) from a metal of work function W is given by: $K=hf-W$.
16. **Heisenberg's Uncertainty Principles**: $\Delta p\Delta x\geq\hbar$ and $\Delta E\Delta t\geq\hbar$
17. The time dependent **Schrödinger Equation**:

$$[-\frac{\hbar^2}{2m}\nabla^2+V]\Psi=i\hbar\frac{\partial\Psi}{\partial t}$$

18. The time independent **Schrödinger Equation**:

$$[-\frac{\hbar^2}{2m}\nabla^2+V(r)]\psi(r)=E\psi(r)$$

19. Probability of **tunneling through a barrier** of width L and of height V_o

$$T=\exp[-2GL] \text{ where } G=\sqrt{\frac{2m(V_o-E)}{\hbar^2}}$$

Question 1 (30 Points)

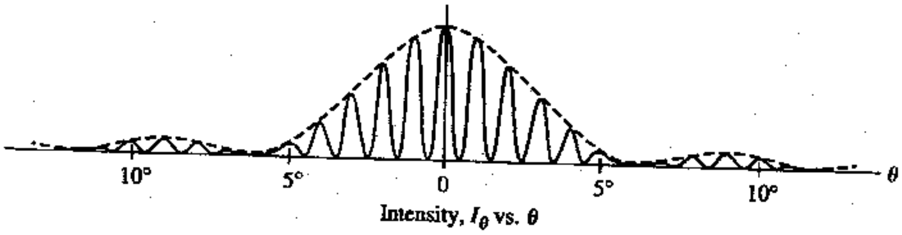


Figure 1(a)

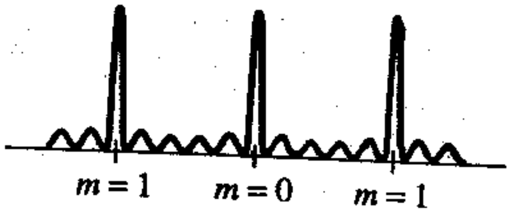


Figure 1 (b)

The diffraction pattern shown in Fig. 1(a) was obtained by shining monochromatic light of wavelength λ=600nm normally on an array of narrow slits of identical width and equally spaced from each other along a line perpendicular to the slits.

(a) (5 Points) How many slits are being illuminated by the light? Give the reasons behind your answer.

(b) (5 Points) Estimate the width of each slit?

(c) (5 Points) Estimate the separation between the slits?

The same light is now incident normally on another array of equally spaced lines forming a diffraction grating. The resultant diffraction pattern is shown in Fig. 1(b). m denotes the order of the diffraction peak.

(d) (5 Points) How many lines of the grating are being illuminated by the light? Give the reasons behind your answer.

(e) (5 Points) Suppose the angular separation between the m=0 and m=1 diffraction peaks is 20°, what is the separation between the lines in the grating?

(f) (5 Points) Suppose the light beam is replaced by an electron beam and the kinetic energy of the electrons is equal to 1 eV and the same diffraction pattern in Fig. 1 (b) is obtained. What should be the separation between the lines in the grating?

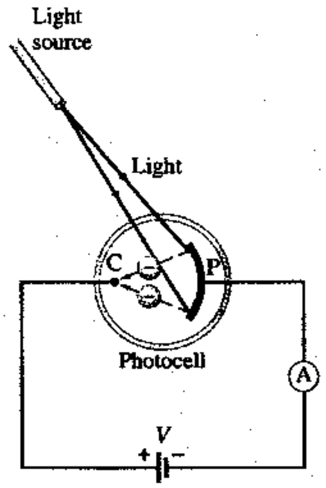
Question 2 (25 Points)

Assume that the sun can be approximated as a black-body radiator.

(a) (3 Points) If the wavelength at which the intensity of the sun's radiation is maximum is 500 nm. Estimate the temperature of the surface of the sun.

(b) (15 Points) Use the Planck's radiation law and the Wien's displacement law to estimate the value of the Planck's constant h. {*Hint*: the transcendental equation e^{-y}=[1-(y/5)] has two roots : y=0 and y~5}

(c) (7 Points) In a photoelectric experiment shown in the following figure it was found that the photocurrent produced by the metal target P, collected by the electrode C and read by the ammeter A is reduced to zero when the potential applied by the battery is equal to ($-V_o$). The value of this stopping potential depends on the wavelength λ of the incident radiation. The value of V_o was found to be 1.4 and 2.254 Volts for λ=400 and 550 nm, respectively. What are the values of the work function W of the metal target and the Planck's constant h determined from this experiment?



Question 3 (total: 20 Points)

When an electron is removed from a He atom a positively charged He^+ ion is formed. In this ion there is only one electron moving around a doubly charged He nucleus so it is very similar to the hydrogen atom except for the fact that the nuclear charge is −2e (where e is the magnitude of the charge of the electron).

(a) (5 Points) Derive the equation which gives the values of $(1/\lambda)$ for the series of emission lines which corresponds to the Balmer series in the hydrogen atom. {*hint*: the Coulomb potential between two charges Q and q separated by a distance r is equal to $(1/4\pi\varepsilon_o)(Qq/r)$}

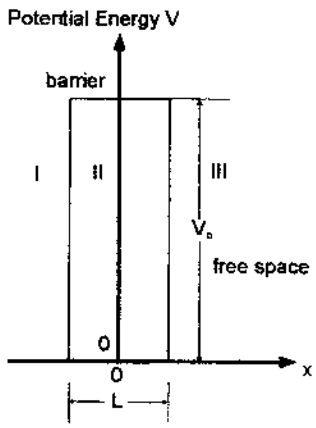
(b) (5 Points) What is the value of the corresponding Bohr radius in the He^+ ion?

(c) (5 Points) What is the potential energy of the electron in the n=2 orbit of the He^+ ion?

(d) (5 Points) What is the kinetic energy of the electron in the n=2 orbit of the He^+ ion?

Question 4 (total: 25 Points)

The following figure shows a one-dimensional potential barrier of height V_o=2 eV and width L=1.5 nm.



A particle with mass *m* equal to that of the electron and kinetic energy E equal to 0.5 eV is incident on the barrier *from the left*.

(a) (5 Points) If we divide the whole space into three regions I, II and III. Write down (you don't have to derive it if you know the answer) the physically acceptable solution to the *time-dependent* Schrödinger equation in all three regions. (Represent the normalization constant as C).

(b) (7 points) Write down (you don't have to derive it if you know the answer) the physically acceptable solutions to the *time-independent* Schrödinger equation in regions I and III. (Represent the normalization constant as A,B,etc).

(c) (3 Points) What are the de Broglie wavelengths for this particle in the regions I and III?

(d) (5 Points) Write down (you don't have to derive it if you know the answer) a physically acceptable solution to the time-independent Schrödinger equation in region II. (Again represent the normalization constant as F).

(e) (5 Points) What is the probability of this particle tunneling through the barrier?

-------------------END OF QUESTIONS-------------------