Physics 7c Final Exam (Muller) 21 May 1999

The following constants may be useful. They are all in MKS units:
$n_W = 1.33$ $n_G = 1.5$ $e = 1.6 \times 10^{-19}$ coul $m_e = 9.1 \times 10^{-31}$ kg $h = 6.7 \times 10^{-34}$ J sec

1. (10 pts) At what angle above the horizontal is the sun, if sunlight reflected from the surface of a calm body of water is completely polarized?

2. (10 pts) Element X is radioactive. At t =1 hour, we have N_1 atoms of element X present. At t = 2 hours, we have N_2 atoms. How many do we expect to find at t = 5 hours?

3. (10 pts) Normally we "explain" alpha particle decay of a nucleus by assuming that the alpha particle was bound inside the nucleus, but had positive energy and tunnelled out. Now I would like you to consider the possibility that in beta decay, that the electron was bound inside the nucleus prior to the decay, and it too tunneled out.

   Estimate the kinetic energy that the electron would have to have to be confined inside the nucleus. Assume that the volume of the nucleus is 10^{-39} cm^3. Give your answer in MeV and compare this to the typical electron kinetic energy found in beta decay.

4. (15 pts) John and Mary are twins. At age zero, Mary takes a trip to a star which is 5 light years away. She travels at 0.6 times the speed of light. John leaves at the same time, but travels at 0.8 times the speed of light. He arrives first, and waits for her to arrive. When she does arrive, what are the ages of John and Mary?

5. This problem is about "resolving power" in microscopes. The numerical aperture N_A of a lens (optical or electron) is defined as the ratio f/D, where f is the focal length, and D is the diameter. You may assume that the minimum possible value of N_A is 1/2.
   (a) (8 pts) In an optical microscope, the objective lens is defined to be the small lens that is close to the object. Let d = the distance between two small objects that can just barely be resolved. Then I can show that $d = x \lambda N_A$, where x is a number. Find x.
   (b) (7 pts) An electron microscope accelerates the electron to an energy of 50 keV (1 keV = 1000 eV). Estimate (get within a factor of 10) the size of the closest pair of objects that can be resolved using an electron microscope with this energy.

6. (10 pts) An automobile is moving at a constant velocity v = 10 meters/sec. The wheels of the automobile have radius R. A tack is stuck on the circumference of one wheel. If you sketch the path of the tack with respect to the road, you will find that it moves in a complicated path known as a "cycloid." Let the acceleration of the tack, at any given moment, be the vector **a**(t). The magnitude of this is $|a(t)|$. What is
   (a) the maximum value for $|a(t)|$ as the car moves forward at constant velocity v?
   (b) the minimum value for $|a(t)|$ as the car moves forward at constant velocity v?
   Hint: this *is* a 7c problem. And you do not have to know anything about cycloids to solve it.

7. (10 pts) A pion is believed NOT to satisfy the Schrodinger equation. Instead, it satisfies the "Klein Gordon equation": $$\hbar^2 \frac{\partial^2 \Psi}{\partial x^2} - \frac{\hbar^2}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = m_\pi{}^2 c^2 \Psi$$
   Assuming that this equation is true, calculate the quantum mechanical phase velocity v_p for the pion in terms of its classical velocity v_c.