University of California, Berkeley **Department of Mechanical Engineering**
ME 106 – Fluid Mechanics **Spring 2009**

Midterm 1

Answer all 3 questions. Hand in the answer sheets only.

QUESTION 1. Suppose that a scalar quantity ϕ, which describes some physical property of a fluid, is specified by

$$\phi = \phi_E(x, y, z, t) = \phi_L(x_0, y_0, z_0, t),$$

where the function ϕ_E is the Eulerian represention and the function ϕ_L is the Lagrangian representation.

(a) Define the material time derivative of ϕ and derive the formula

$$\frac{D\phi}{Dt} = \frac{\partial \phi}{\partial t} + \nabla\phi \cdot \boldsymbol{V},$$

where $\boldsymbol{V} = \boldsymbol{V}(x, y, z, t)$ is the velocity field of the fluid.

(b) Suppose that the density and velocity fields in a fluid are given by

$$\rho = \rho_0 + txy^2, \quad \boldsymbol{V} = y\sin(2t)\boldsymbol{i} + z^2(\boldsymbol{j} + \boldsymbol{k}),$$

where $\rho_0(x_0, y_0, z_0)$ is the density of the fluid at time $t = 0$. Calculate $D\rho/Dt$.

QUESTION 2. For a fluid in motion, let

$$\boldsymbol{n}(x, y, z, t) = n_x\boldsymbol{i} + n_y\boldsymbol{j} = \cos\theta\,\boldsymbol{i} + \sin\theta\,\boldsymbol{j}$$

be the outward unit normal vector to a surface passing through the point (x, y, z) at time t. Recall the stress vector formula

$$\boldsymbol{t}(x, y, z, t, \boldsymbol{n}) = \boldsymbol{t}(x, y, z, t, \boldsymbol{i})\, n_x + \boldsymbol{t}(x, y, z, t, \boldsymbol{j})\, n_y.$$

(a) Describe in words and with a diagram what $\boldsymbol{t}(x, y, z, t, \boldsymbol{i})$ represents physically.

(b) If at some spatial point (x, y, z) and time t, the stress vectors $\boldsymbol{t}(x, y, z, t, \boldsymbol{i})$ and $\boldsymbol{t}(x, y, z, t, \boldsymbol{j})$ have the values

$$\boldsymbol{t}(x, y, z, t, \boldsymbol{i}) = [-200\,\boldsymbol{i} + 150\,\boldsymbol{j} + 80\,\boldsymbol{k}] \quad \mathrm{KPa}\,, \quad \boldsymbol{t}(x, y, z, t, \boldsymbol{j}) = [150\,\boldsymbol{i} - 200\,\boldsymbol{j}] \quad \mathrm{KPa}\,,$$

(i) calculate the three components of the stress vector acting on a surface that passes through (x, y, z) at time t and has an outward unit normal vector $\boldsymbol{n}$ that lies in the xy-plane and makes an angle of $\pi/6$ radians with the positive x-axis; ***(ii)*** calculate the normal stress N and the shearing stress τ for the surface in ***(i)***.

QUESTION 3. Consider an inviscid fluid with constant density, and suppose that it is free from body forces. Recall Euler's hydrodynamical equation

$$-\boldsymbol{\nabla} p = \rho \boldsymbol{a} \qquad \mathrm{N/m^3}$$

(a) If the fluid is spinning with constant angular about the z-axis in a closed container (in a gravity-free environment), it is known that the acceleration field may be expressed as

$$\boldsymbol{a} = -\frac{c}{\rho}(x\boldsymbol{i} + y\boldsymbol{j}) \qquad \mathrm{m/s^2}\,.$$

where c is a positive constant. Solve for the pressure field.

(b) If the pressure has a constant value p_0 along the z-axis, calculate its value on the cylindrical surface

$$x^2 + y^2 = 1 \qquad \mathrm{m^2}\,.$$

ANSWER TO QUESTION 1 **NAME:________________________________**

ANSWER TO QUESTION 2 **NAME:________________________________**

(Question 2, Continued)

ANSWER TO QUESTION 3

NAME:________________________________