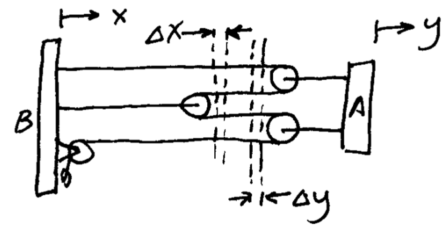
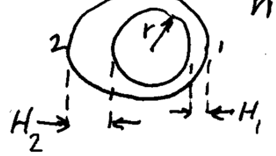
SOLUTIONS – TEST #1
ME 104, S'04

(2)



$$-4\Delta x + 4\Delta y + 8\Delta t = 0$$
$$-4\dot{x} + 4\dot{y} + 8 = 0$$
$$\dot{x} = \dot{y} + 2 \Rightarrow \boxed{\dot{x} = 10\,m/s}$$

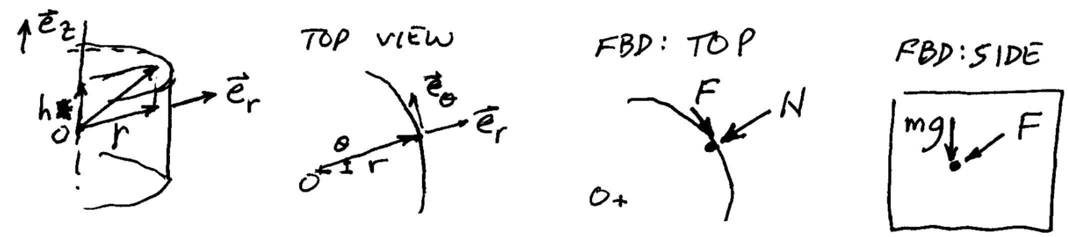
(3)



$h = rV_\theta =$ CONSTANT. $r_1 V_{\theta_1} = r_2 V_{\theta_2}$

$$(r+H_1)4V_{\theta_2} = (r+H_2)V_{\theta_2} \Rightarrow \boxed{H_2 = 4H_1 + 3r}$$
$$\boxed{H_2 = 20.7 \times 10^6\, m}$$

(4)



(a) CONSTRAINTS: $r =$ CONSTANT HENCE $\dot{r} = \ddot{r} = 0$. SLIDING FRICTION IMPLIES $F = \mu N$ WHERE N IS NORMAL FORCE. $\vec{F} = -\dfrac{\mu N \vec{V}}{\sqrt{V_\theta^2 + V_z^2}}$

(b) $\vec{r}_{A/O} = r\vec{e}_r + h\vec{e}_z$, $\vec{V}_A = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta + \dot{h}\vec{e}_z$, $\vec{a}_A = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta + \ddot{h}\vec{e}_z$

$$m\left[-r\dot{\theta}^2\vec{e}_r + r\ddot{\theta}\vec{e}_\theta + \ddot{h}\vec{e}_z\right] = -mg\vec{e}_z - N\vec{e}_r - \frac{\mu N(r\dot{\theta}\vec{e}_\theta + \dot{h}\vec{e}_z)}{\sqrt{r^2\dot{\theta}^2 + \dot{h}^2}}$$

$\vec{e}_r$: $mr\dot{\theta}^2 = N$ (1)

$\vec{e}_\theta$: $mr\ddot{\theta} = \dfrac{-\mu N r\dot{\theta}}{\sqrt{r^2\dot{\theta}^2 + \dot{h}^2}}$ (2)

$\vec{e}_z$: $m\ddot{h} = -mg - \dfrac{\mu N \dot{h}}{\sqrt{r^2\dot{\theta}^2 + \dot{h}^2}}$ (3)

(1), (2) $\Rightarrow$ $\ddot{\theta} = \dfrac{-\mu r\dot{\theta}^3}{\sqrt{r^2\dot{\theta}^2 + \dot{h}^2}}$

(1), (3) $\Rightarrow$ $\ddot{h} = -g - \dfrac{\mu r\dot{h}\dot{\theta}^2}{\sqrt{r^2\dot{\theta}^2 + \dot{h}^2}}$

LET $y_1 = \theta$, $y_2 = \dot{\theta}$, $y_3 = h$, $y_4 = \dot{h}$

$\dot{y}_1 = y_2$

$\dot{y}_2 = \dfrac{-\mu r y_2^3}{\sqrt{r^2 y_2^2 + y_4^2}}$

$\dot{y}_3 = y_4$

$\dot{y}_4 = -g - \dfrac{\mu r y_4 y_2^2}{\sqrt{r^2 y_2^2 + y_4^2}}$