1/ (a) $\dfrac{Du}{Dt} = \dfrac{\partial u_E}{\partial t} + \underline{V} \cdot \nabla u_E$

$$\boxed{\frac{Du}{Dt} = \frac{\partial u_E}{\partial t} + u_E \frac{\partial u_E}{\partial x} + v_E \frac{\partial u_E}{\partial y}}$$

$\dfrac{Dv}{Dt} = \dfrac{\partial v_E}{\partial t} + \underline{V} \cdot \nabla v_E$

$$\boxed{\frac{Dv}{Dt} = \frac{\partial v_E}{\partial t} + u_E \frac{\partial v_E}{\partial x} + v_E \frac{\partial v_E}{\partial y}}$$

(b) $\underline{\omega} = \text{curl } \underline{V} = \left(\dfrac{\partial v_E}{\partial x} - \dfrac{\partial u_E}{\partial y} \right) \underline{k}$

(c) Incompressible $\longrightarrow$ no change in volume with time

$\therefore \dfrac{1}{\Delta V} \dfrac{D(\Delta V)}{Dt} = 0 \longrightarrow \boxed{\underline{\nabla} \cdot \underline{V} = 0 \quad \text{or} \quad \dfrac{\partial u_E}{\partial x} + \dfrac{\partial v_E}{\partial y} = 0}$

(d) Streamline: a line that is everywhere tangent to the velocity vectors (instantaneously).

$\therefore$ Along a streamline, the vectors

$\underline{V} = u\underline{i} + v\underline{j}$ and $d\underline{r} = dx\underline{i} + dy\underline{j}$ will be parallel.

If parallel, $\underline{V} \times d\underline{s} = \underline{0}$

$$\det \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ u & v & 0 \\ dx & dy & 0 \end{vmatrix} = \underline{k}\,(u\,dy - v\,dx) = \underline{0}$$

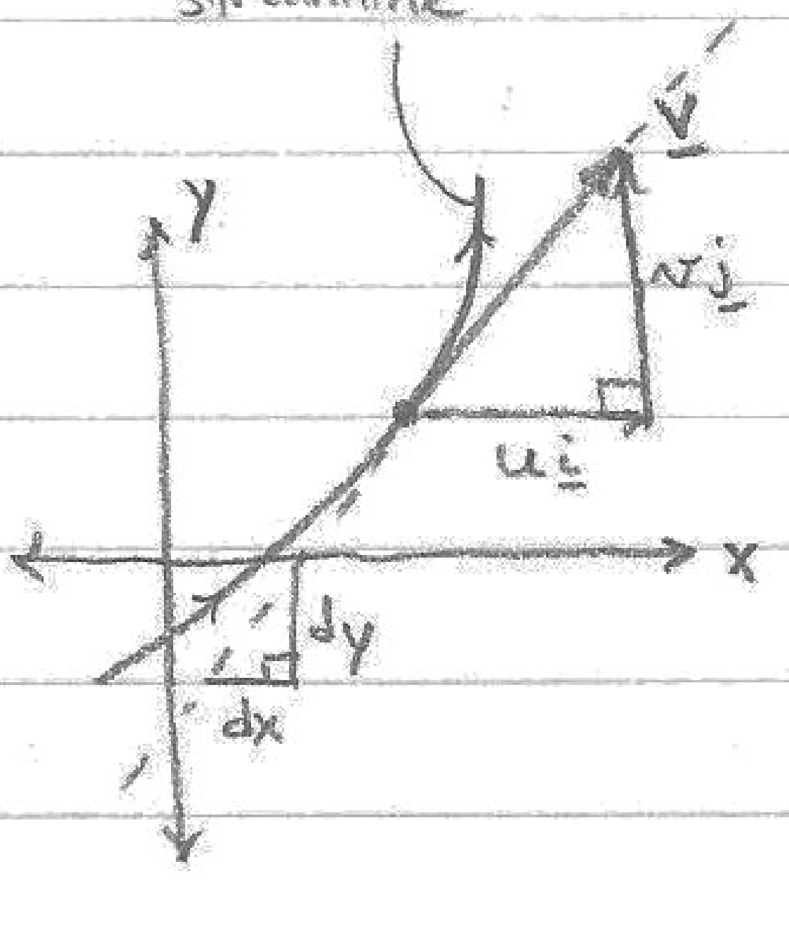
$$u\,dy - v\,dx = 0$$

$$u\,dy = v\,dx$$

$$\boxed{\frac{dy}{dx} = \frac{v}{u}}$$

(2)

Graphically:



Since the velocity is tangent to the streamline, the ratios of $\dfrac{dy}{dx}$ and $\dfrac{v}{u}$ must be equal $\rightarrow \boxed{\dfrac{dy}{dx} = \dfrac{v}{u}}$

(3)

Along streamline $\underline{V} \parallel d\underline{s} \rightarrow \underline{V} = \xi d\underline{s}$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \xi \begin{bmatrix} dx/ds \\ dy/ds \end{bmatrix} \longrightarrow u\underline{i} + v\underline{j} = \xi\left(\frac{dx}{ds}\underline{i} + \frac{dy}{ds}\underline{j}\right)$$

$$u\underline{i} + v\underline{j} = \xi\frac{dx}{ds}\left(\underline{i} + \frac{ds}{dx}\frac{dy}{ds}\underline{j}\right)$$

$$u\underline{i} + v\underline{j} = \xi\frac{dx}{ds}\left(\underline{i} + \frac{dy}{dx}\underline{j}\right)$$

$$\therefore \frac{v}{u} = \frac{\cancel{\xi}\cancel{\frac{dx}{ds}}\frac{dy}{dx}}{\cancel{\xi}\cancel{\frac{dx}{ds}}} = \underline{\underline{\frac{dy}{dx}}}$$

$u = -a^2 c y, \quad v = b^2 c x$

2/ a) $\operatorname{div} \underline{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0 \quad \therefore$ isochoric

b) $\underline{\omega} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\underline{k} = \boxed{c\left(b^2 + a^2\right)\underline{k} \neq 0} \Rightarrow$ rotational

c) $\underline{a} = \cancel{\frac{\partial \underline{v}}{\partial t}} + \nabla \underline{v} \cdot \underline{v} = \underline{0} + \begin{bmatrix} 0 & -a^2 c \\ b^2 c & 0 \end{bmatrix} \begin{pmatrix} -a^2 c y \\ b^2 c x \end{pmatrix}$

$\boxed{\underline{a} = -a^2 b^2 c^2 \left(x \underline{i} + y \underline{j}\right)}, \quad \Rightarrow \quad k = a^2 b^2 c^2$

d) $-\nabla P = \rho \underline{a}$

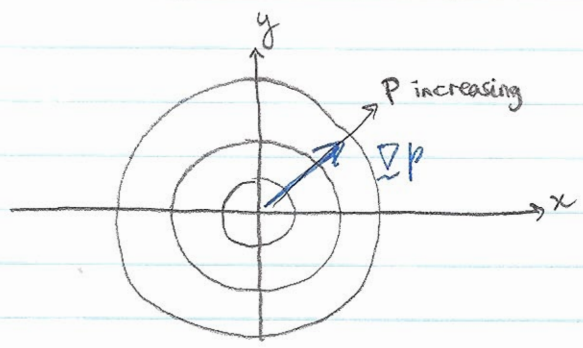
$\rightarrow \begin{cases} \frac{\partial P}{\partial x} = +\rho a^2 b^2 c^2 x \Rightarrow P = \rho a^2 b^2 c^2 \frac{x^2}{2} + f_1(y) + K \\ \frac{\partial P}{\partial y} = +\rho a^2 b^2 c^2 y \Rightarrow P = \rho a^2 b^2 c^2 \frac{y^2}{2} + f_2(x) + K \end{cases}$

$\rightarrow P(x,y) = \frac{\rho a^2 b^2 c^2}{2}\left(x^2 + y^2\right) + K'$

BC: $P(0,0) = P_0 \rightarrow K' = P_0$

$\rightarrow P(x,y) = \frac{\rho a^2 b^2 c^2}{2}\left(x^2 + y^2\right) + P_0$

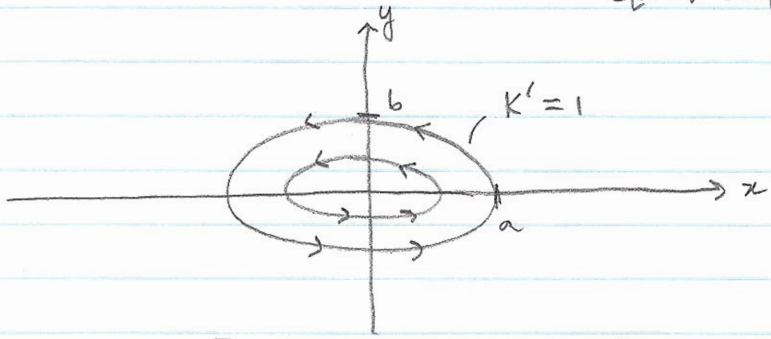
$r^2 = x^2 + y^2 \quad \therefore$ as $r\uparrow, \ P\uparrow$



2/ e) $\frac{dy}{dx} = \frac{v}{u} \Rightarrow \frac{dy}{dx} = \frac{b^2 c x}{-a^2 c y} \Rightarrow b^2 x\,dx + a^2 y\,dy = 0$

$\rightarrow \frac{b^2}{2} x^2 + \frac{a^2}{2} y^2 = K \Rightarrow \boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = K' = \text{const}}$

eqn of ellipse



f) $B(x,y) = \left[\frac{\rho a^2 b^2 c^2}{2}(x^2+y^2) + P_0 \right] + \frac{1}{2}\rho\left(a^4 y^2 + b^4 x^2\right) c^2$

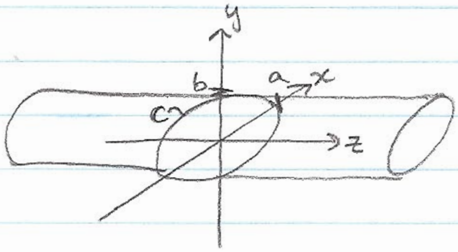
g) $B(a,0) = \frac{\rho a^2 b^2 c^2}{2} a^2 + P_0 + \frac{1}{2}\rho b^4 a^2 c^2 = \text{const}$

$B(0,b) = \frac{\rho a^2 b^2 c^2}{2} b^2 + P_0 + \frac{1}{2}\rho a^4 b^2 = \text{const}$

$B(a,0) = B(0,b) = \text{const}$, as expected since both points lie along the same streamline.

h) No. $\underline{\omega} \neq 0$ therefore can only apply BE <u>along</u> streamlines

i)



j) $\Gamma = \int_A (\nabla \times \underline{V}) \cdot \underline{n}\, dA = \int_A \underline{\omega} \cdot \underline{n}\, dA = c(a^2+b^2)\int_A dA$

$\boxed{\Gamma = \pi abc\,(a^2+b^2)}$