CS-172 Quiz 1 February 12, 1993
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Each of the following questions counts equally. Try to keep your answers succinct.

> **Pumping Lemma:**
> **If** L is regular **then**
> $(\exists n)(\forall z \in L,\ |z| \geq n)(\exists uvw$ such that $z = uvw$ and $|uv| \leq n$ and $|v| \geq 1)(\forall i) : uv^iw \in L$.

1. Let language L be given by the regular expression 10^*1.
   (a) Construct a DFA accepting L.
   (b) Construct a DFA accepting $\overline{L}$.
   (c) Construct a regular expression for $\overline{L}$. If your expression is complicated, you should be able to give a succinct overview in english to convince me that your expression is correct.

2. We wish to prove that $L = \{0^i1^j : \gcd(i,j) = 1\}$ is not regular. Recall that $\gcd(i,j) = 1$ if i and j have no factors in common. So, $0^{10}1^3 \in L$, $0^51^5 \notin L$ and $0^61^{10} \notin L$. Here are three "proofs" that L is not regular, one correct. Identify the correct proof with a $\bigstar$, and succinctly explain what is wrong with the other two proofs. (Hint: The incorrect proofs use the pumping lemma wrong — nothing is wrong with the algebra.)
   (a) It suffices to show that $\overline{L}$ is not regular. Fix n in the pumping lemma. If $z = 0^p1^p \in \overline{L}$ for prime $p > n+1$ and let $z = uvw$ as in the lemma. No matter what uvw, $uw = 0^l1^p$ for some $1 < l < p$ and $\gcd(l,p) = 1$. So $uw \notin \overline{L}$ and $\overline{L}$ is not regular.
   (b) Fix n in the pumping lemma. Note that consecutive numbers above 1 cannot have a common factor. So the string $z = 0^{2n+1}1^{2n} \in L$. Choose $z = uvw$ as in the lemma, where $v = 0^l$ for l odd. $uw = 0^{2n+1-l}1^{2n} \notin L$ since both $2n+1-l$ and $2n$ are divible by 2. Hence L is not regular.
   (c) Fix n in the pumping lemma. Without loss of generality, $n \geq 2$. Choose $z = 0^n1^n \notin L$ and fix $z = uvw$. Now v must be of the form 0^l, $1 \leq l \leq n$. Then $uv^{n+1}w = 0^{n+ln}1^n \notin L$ since both $n + ln$ and n are divisible by n. Hence, L is not regular.