CS 172, Fall 1999
Midterm
Professor M. Jordan

172 students:

On this exam, I am more interested in testing your ability in setting up the solution to a variety of problems than necessarily in getting all of the details. I don't expect anyone to answer all of the questions fully and correctly.

We will be reasonably generous with partial credit, so it would be better to provide partial answers to many questions than complete answers to a few questions. For the more difficult questions, you might consider first writing down a brief section called "Sketch of answer," and later, if you have time, writing down a more detailed section called "Details."

--Mike

Problem #1

Closure properties of languages associated with Turing machines.
(a) Prove that the union of two Turing-recognizable languages is Turing recognizable.
(b) Prove that the union of two decidable languages is decidable.

Problem #2

Show that the class of context-free languages is closed under *. (*Hint*: It is probably easiest to do this with a grammer, where a fairly simple construction suffices, but it is doable with PDA's if you prefer).

Problem #3

(a) Draw an NFA that recognizes the languages:
A = {w element of {a,b,c}* | w contains at least two b's or at least one c}.
(b) Give a regular expression that describes this language.

Problem #4

Design a PDA for
L = {(0^i)(1^j) | i does not equal j and i, j >= 0}.
A high-level English description will get yo partial credit, and a diagram will receive full credit.

Problem #5

Let sigma = {0,1,...,9}. Let
L = { | M is a DFA and M does not accept any string containing 555 as a substring}.
Show that L is decidable. (*Hint*: Use the fact that it is possible to contruct a DFA that recognizes the regular language sigma*555sigma*. Also use the fact that regular languages are closed under intersection.).

Problem #6

Let A and B be Turing-reconizable languages. Let (A intersection B) and (A union B) be decidable. Show that A and B are decidable. (*Hint*: Use a Venn diagram and analyze the decidability of various regions of the diagram).

Problem #7

Consider the problem of testing whether a two-tape Turing machine ever writes a nonblank symbol on its second tape. Formulate this problem as a language. Show that this language is undecidable. (*Hint*: Use a reduction from (A)tm. The basic idea is to contruct a two-tape machine that simulates a Turing machine M on string w.).
