Name. 1

CS-172 David Wolfe Quiz 2 March 12, 1993

Each of the following questions counts equally. Try to keep your answers succinct.

1. Define p_x to be the parity bit of $x \in \{0,1\}^*$.

$$p_x = \begin{cases} 0 & \text{if } x \text{ has an even number of 1's} \\ 1 & \text{if } x \text{ has an odd number of 1's} \end{cases}$$

Design a Turing machine, M, with input alphabet $\{0,1\}$ to compute the function $f(x) = xp_x$. For example, f(011) = 0110 and f(010) = 0101. You'll receive:

- 2 points: if your machine properly appends the parity bit onto x.
- 3 points: if, additionally, your machine accepts with the head at the start of the tape. (It should make it's last transition to the first tape location while entering an accept state.)
- 4 points: if, in addition, your machine uses no extra tape alphabet symbols.

Specify the entire machine using proper notation. You may not use any extentions of the Turing machine model (such as a 2-way infinite tape), nor can the machine recognize start of tape.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

You may specify the transition function with either a table or (to make grading easier) a state diagram which marks transition arrows with X/YD, where $\delta(q,X) = (q',Y,D)$ for symbols X and Y and direction D. (My solution used 9 states. Feel free to use a few more, but your solution should be clear.)

- 2. Let $L = \{\langle M \rangle : |L(M)| = 1\}$. (I.e., M accepts exactly one string.) Consider the following reduction: $f(\langle M, w \rangle) = \langle M' \rangle$, where M'(x) accepts if $x = \epsilon$ or if M(w) accepts. In all other cases, M' rejects. Recall that $L_u = \{ \langle M, w \rangle : w \in L(M) \}.$
 - (a) (2 points) If M accepts w, what is L(M')? How about if M does not accept w?
 - (b) (2 points) Does f reduce $L_u \alpha L$, $\overline{L_u} \alpha L$, $L \alpha L_u$, $\overline{L} \alpha L_u$? (It does one of the four.) Explain. (If you're stuck on this part, you may ask for the solution. You'll lose credit for the part.)
 - (c) (2 points) Given your answer above, what can we conclude about L?
 - (d) (2 points) What needs to be checked to verify that f is, in fact, a reduction.
 - (e) (2 points) Prove that f is a reduction.