CS 172 — Fall 1997	**Prelim 2**
Computability and Complexity	**November 5, 1997**

Consider the following four languages:

- $A_1 = \{\#0^n 1^{2n} 0^n \# \mid n \geq 0\}$
- $A_2 = \{\#0^m 1^{m+n} 0^n \# \mid m, n \geq 0\}$
- $A_3 = \{\langle M \rangle \mid M$ is a DTM that for every input of length n uses at most n steps$\}$
- $A_4 = \{\langle M \rangle \mid M$ is a DTM that for every input of length n uses at most n tape cells$\}$

DTM stands for deterministic Turing machine. If a Turing machine uses at most n steps, then it must reach an accepting or rejecting state within at most n steps. If a Turing machine uses at most n tape cells, then it may loop without moving past the first n tape cells.

We say that a language A is *co-c.f.* if its complement $\overline{A}$ is context-free. Consider the following six mutually exclusive statements about a language A:

S1 The language A is regular.
S2 The language A is context-free and co-c.f., but not regular.
S3 The language A is context-free, but not co-c.f.
S4 The language A is co-c.f., but not context-free.
S5 The language A is recursive, but neither context-free nor co-c.f.
S6 The language A is r.e., but not recursive.
S7 The language A is co-r.e., but not recursive.
S8 The language A is neither r.e. nor co-r.e.

You are asked to determine for each language A_1 to A_4 which one of the statements **S1** to **S8** is true. You need to justify your answers as follows:

- To justify **S1**, give the transition diagram of a finite automaton that accepts A.
- To justify **S2**, give the transition diagram of a *deterministic* pushdown automaton that accepts A. You need not give a proof that A is not regular.
- To justify **S3**, give (i) a context-free grammar that generates A and (ii) a pumping proof that $\overline{A}$ is not context-free.
- To justify **S4**, give (i) a context-free grammar that generates $\overline{A}$ and (ii) a pumping proof that A is not context-free.
- To justify **S5**, give (i) a high-level description of a Turing decider that accepts A, (ii) a pumping proof that A is not context-free, and (iii) a pumping proof that $\overline{A}$ is not context-free.
- To justify **S6**, give (i) a high-level description of a Turing machine that accepts A and (ii) a mapping reduction from either TmMembership or $\overline{\text{TmEmptiness}}$ to A.
- To justify **S7**, give (i) a high-level description of a Turing machine that accepts the complement of A and (ii) a mapping reduction from $\overline{\text{TmMembership}}$ or TmEmptiness to A.
- To justify **S8**, give a mapping reduction from TmUniversality or $\overline{\text{TmUniversality}}$ to A.

Good luck!