UNIVERSITY OF CALIFORNIA, BERKELEY — College of Engineering

Statics (E36)

Final Examination

Problem 1. (20 points)
Draw the shear and moment diagrams for the beams shown in Figure 1 :
(A) $w_0 = 10N/m$, $a = 1.0m$:
(a1) Find the reactions, (a2) Find the expressions for $V(x)$ and Draw the shear diagram, and (a3) Find the expressions for $M(x)$ and Draw the moment diagram ;

(B) $M = 10$ N-m and $L = 5m$.:
(b1) Find the reactions, (b2) Find the expressions for $V(x)$ and Draw the shear diagram, and (b3) Find the expressions for $M(x)$ and Draw the moment diagram .

Hints:

$$\frac{dV}{dx} = -w(x), \quad \text{and} \quad \frac{dM}{dx} = V(x) .$$

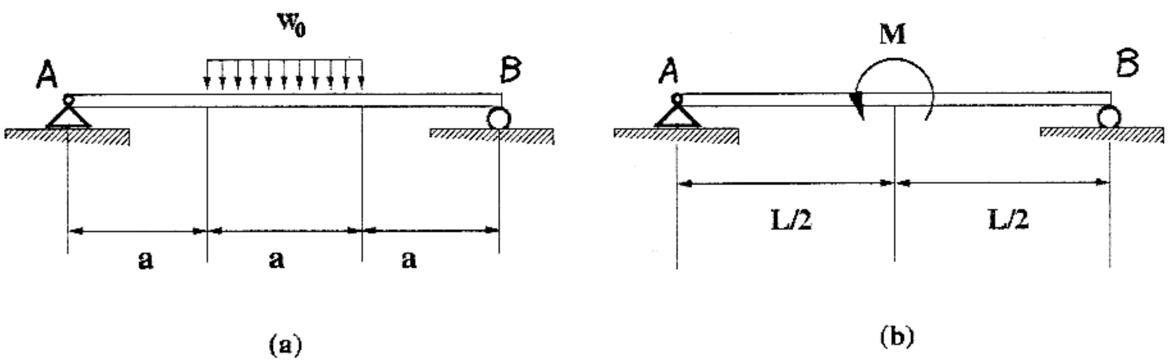


Figure 1: A simply supported beam with different load conditions

Problem 2. (15 points)
(1) Find the centroid of the cross-section shown in Figure 2 and set up the centroidal axes.

$$\bar{y} = \frac{\sum_i \bar{y}_i A_i}{\sum A_i}$$

(2) Find I_x with respect to the global centroidal axes.

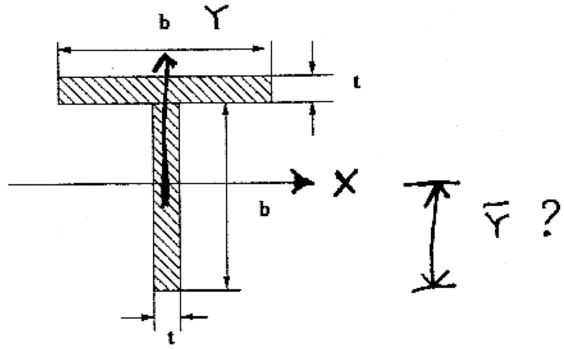


Figure 2: The cross section of a T-beam

Hints:

$$I_x = \int_A y^2 dA, \quad I_y = \int_A x^2 dA \tag{1}$$

$$I_x = (I_x)_c + d^2 A, \quad \Leftarrow \text{ Parallel Axis Theorem} \tag{2}$$

$$I_{rectangular} = \frac{bh^3}{12} \quad \Leftarrow \text{ (The genetic formula for local centroidal axis)} \tag{3}$$

Problem 3 (15 points)

A 2.4-m-long boom is held by a ball-and-socket joint at C and by two cables AD and BE. An external load $\mathbf{W} = -880\mathbf{j}$ is acting at point A. Determine the tension in each cable and the reaction at the point C.

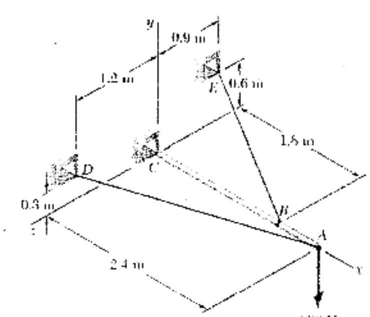


Fig. P4.116

Figure 3: A three-dimensional structure

(a) Draw free-body diagram for bar AC;

(b) Write down the vector expressions for $\mathbf{r}_A$, $\mathbf{r}_B$, $\mathbf{r}_{AD} = \mathbf{r}_D - \mathbf{r}_A$, and $\mathbf{r}_{BE} = \mathbf{r}_E - \mathbf{r}_B$;

(c) Find the forces in the vector form, $\mathbf{W}$, $\mathbf{T}_{AD}$, and $\mathbf{T}_{BE}$;

(d) Write the vector form moment equilibrium equation,

$$\sum \mathbf{M}_C = \sum_i \mathbf{r}_{ic} \times \mathbf{F}_i = 0$$

and find T_{AD} and T_{BE};

(f) Write the vector form force equilibrium equation

$$\sum_i \mathbf{F}_i = 0$$

and find C_x, C_y, and C_z

Hint:

$$\mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

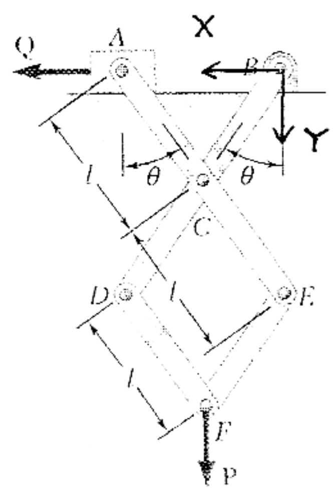


Fig. P10.50 and P10.51

Problem 4. (20 points)
Denoting by μ_s the coefficient of static friction between the block attached to rod ACE and the horizontal surface, derive expressions in terms of P, μ_s, and θ for the largest and smallest magnitudes of the force **Q** for which equilibrium is maintained.

(a) Draw the free-body diagram for the whole system;

(b) Find the ground support force A_y and the friction force acting on block A;

(c) Find x_A, y_F and the virtual displacements δx_A and δy_F;

(d) Write down δU and let $\delta U = 0$ to find Q_{max} and Q_{min};

Figure 4: Friction and Virtual Work Method

Problem 5. (15 points)
A floor truss is loaded as shown. Determine the forces in members FI, HI, and HJ.

(1) Find the reactions at the point A and the point K;

(2) Use method of section making a cut, draw the free-body diagram of the remaining substructure, and then solve for internal forces F_{FI}, F_{HI}, and F_{HJ} .

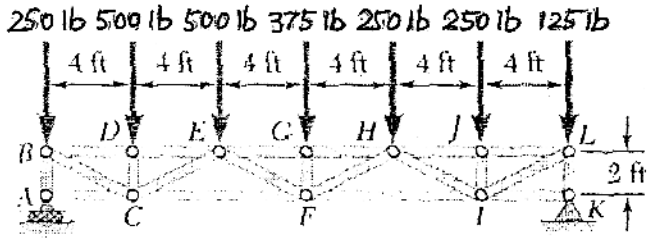
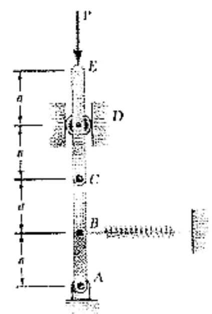


Figure 5: A Truss System with External Loads.



Problem 6. (15 points)
Bar AC is attached to a hinge at A and to a spring at the point B. The spring constant is k, and it is undeformed when the bar is vertical. Find the range of values of P for which the equilibrium of the system is stable at shown the position $\theta = 0$.
(a) Find x_c in terms of θ and ϕ and find the relationship between θ and ϕ when $\theta, \phi << 1$;
(b) Find x_B and y_E and write down the potential function for the system in terms of θ, P, k and a;
(2) Show that $\theta = 0$ is an equilibrium position by using the equilibrium condition

$$\frac{dV}{d\theta} = 0 \ ;$$

(3) Find the range of values of P such that the equilibrium at $\theta = 0$ is stable $\frac{d^2V}{d\theta^2} > 0$.

SOLUTION

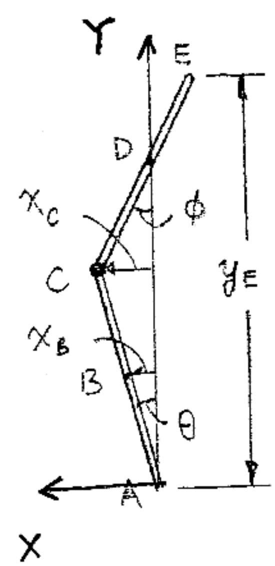


Figure 6: Equilibrium of a two-bar system.