Ö. Savaş NAME: **SOLUTIONS** SID:

100 Lewis Hall, 8:00-11:00, Friday, May 11, 2007

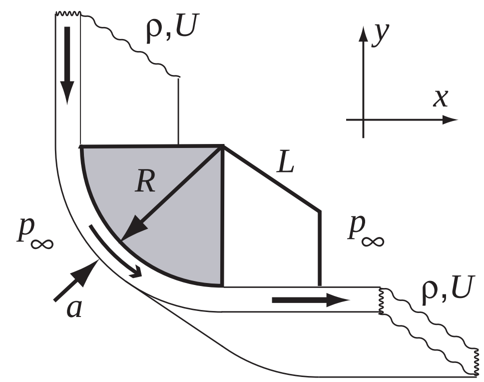
ME 106
FLUID MECHANICS
FINAL EXAM – open book, notes

grading chart

1(20)	2(20)	3(20)	4(20)	5(20)	6(20)	total (120)

YOU MUST SHOW ALL WORK, AND PRESENT ALL ANSWERS IN DIMENSIONALLY CORRECT FROM. FAILURE TO COMPLY WILL SUBSTANTIALLY REDUCE YOUR GRADE.

1.(20% - no partial credit) Suppose a thin sheet of water is skimming a quarter-cylindrical shell of radius R. The sheet has a thickness of $a \ll R$, a width of L and wraps the shell. Assume uniform density ρ and flow speed U throughout the sheet. Ignore gravity, viscosity, and surface tension. Note that pressure is atmospheric on all open surfaces in contact with air.



(a) Determine the pressure distribution over the curved surface.

(b) Determine the force exerted on the shell by the water sheet.

SOLUTION

(a)

$$\frac{dp}{dn} \approx \frac{\Delta p}{a} = \rho \frac{U^2}{R} \Longrightarrow \Delta P = \rho a U^2/R, \quad \text{uniform pressure}$$

(b) Force on fluid

$$\mathbf{F}_f = \int_S \rho \mathbf{u}(\mathbf{u} \cdot d\mathbf{A}) = \rho(U,0)\,[(U,0)\cdot(+A,0)] + \rho(0,-U)\,[(0,-U)\cdot(0,+A)] = (+\rho U^2 A,\, +\rho U^2 A)$$

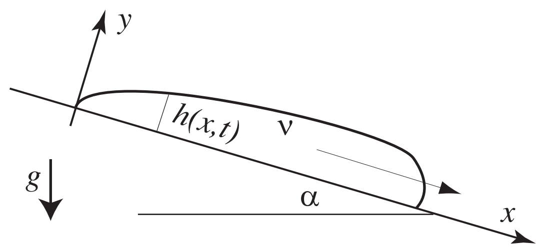
where $A = aL$ is the cross-sectional area of the water sheet.

Force on shell

$$\mathbf{F}_s = -\mathbf{F}_f = (-\rho U^2 A,\, -\rho U^2 A) = \rho U^2 A(-1,\, -1)$$

2.(20%) Consider the spreading of a two-dimensional liquid blob (strip of liquid) on an incline under gravity. The thinning shape of the back side seems to show, in long time, an asymptotic behavior which depends on the viscosity ν, the angle α; that is

$$h = h(x, t; \nu, g, \alpha)$$



Ignore surface tension. Based on dimensional analysis, how many dimensionless numbers are relevant to this problem. Determine all of them. (4+16)

Extra Credit(20%): Based on your intuition, your experience in boundary layers, and that the flow is a balance between viscous diffusion and convection, can you guess the (similarity) solution?

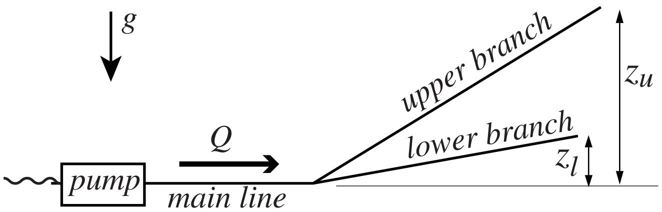
Answer:

$$\left(\Pi_1, \Pi_2, \Pi_3, \Pi_4\right) = \left(\frac{h}{x}, \frac{\nu t}{x^2}, \frac{x}{gt^2}, \alpha\right)$$

Analytic solution:

$$h = \sqrt{\frac{\nu}{g\, sin\alpha}}\sqrt{\frac{x}{t}} \quad \Longrightarrow \quad \Pi_1 = \sqrt{\frac{\Pi_2 \Pi_3}{sin\Pi_4}}$$

3.(20%) Determine the power needed to pump $Q = 30$ liters/second water from the reservoir in the figure. Inlet pressure to the pump and the discharge pressure at the ends of the branches are all atmospheric. All pipes are smooth. How much would the pumping power be reduced if the discharges were slowed down to nearly stagnant state using perfect diffusers. Ignore all minor losses. (15+5)



Main line: 10cm diameter, 300m-long.

Lower branch: 5cm diameter, 250m-long, discharging at 20m elevation.

Upper branch: 5cm diameter, 200m-long, discharging at 60m elevation.

Equal flow rates in the branches, hence the same Re throughout.

$$U = \frac{4Q}{\pi d^2} \qquad Re = \frac{Ud}{\nu} = \frac{4Q}{\pi \nu d}$$

$$\frac{p_1}{\rho} + \frac{1}{2}U_1^2 + gz_1 = \frac{p_2}{\rho} + \frac{1}{2}U_2^2 + gz_2 + h_l \qquad h_l = f \times \frac{L}{d} \times \frac{U^2}{2}$$

	Main Line Smooth pipe	Upper Branch Smooth pipe	Lower Branch Smooth pipe
L (m)	300	200	250
D(m)	0.10	0.05	0.05
$\epsilon = e/D$	0	0	0
z(m)	0	60	20
$Q(m^3/s)$	0.030	0.015	0.015
$u(m/s) = 4Q/\pi d^2$	3.82	7.64	7.64
$Re = Ud/\nu(10^5)$	3.82	3.82	3.82
f	0.0135	0.0135	0.0135
$[\frac{f}{2g}\frac{L}{D}u^2 + z]$ (m)	30.1	**220.7**	**220.8**

$$\dot{W} = \dot{m}\Delta p_{pump} = \dot{m}(\Delta p_m + \Delta p_{a\ branch}) = \rho q_m g(\Delta h_m + \Delta h_{a\ branch}) = \rho q_m g(30.1 + 220.7) = 74kW$$

$$\dot{W}_{diff} = \rho(q_u \times u_u^2 + q_l \times u_l^2)/2 = 0.88kW$$

4.(20%) Consider a wind surfer on the bay who is sailing a rig with $9.0\,m^2$ sail area, approximately 2.0 meters wide and 4.5 meters high. The surfer feels a head wind speed of about 8 m/s. Estimate the skin friction drag on the sail assuming (a) laminar flow, (b) turbulent flow. Estimate the laminar boundary layer thickness at the middle of the sail. (For reference, the world wind surfing speed record is 25 m/s set in 2005.) (8+8+4)

$$Re_L = \frac{UL}{\nu} = \frac{800 \times 200}{0.15} = 1.07 \cdot 10^6$$

(a) Laminar Flow (Equation 135 or 143)

$$C_D = 1.328 Re_l^{-0.5} = 1.328 \times (1.07 \cdot 10^6)^{-0.5} = 1.29 \cdot 10^{-3} \quad \Longrightarrow$$

Considering both sides of the sail

$$2D = 2 \times \left(\frac{1}{2}\rho U^2 \times A \times C_D\right) = 2 \times \left(\frac{1}{2} \times 1.205 \times 8^2 \times 9 \times 1.29 \cdot 10^{-3}\right) N = 2 \times (0.45)\, N = 0.9\, N$$

Laminar boundary layer thickness at mid-point

$$\delta = 5.0\sqrt{\nu x/U} = 5 \times \sqrt{0.15 \times 100/800}\ cm = 0.7\, cm$$

(b) Turbulent Flow (Equation 144)

$$C_f = 0.074\, Re_l^{-1/5} = 0.074 \times (1.07 \cdot 10^6)^{-1/5} = 4.61 \cdot 10^{-3} \quad \Longrightarrow$$

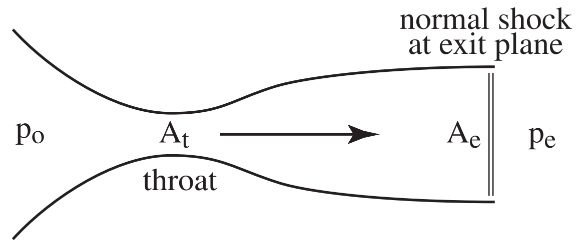
$$2D = 2 \times \left(\frac{1}{2}\rho U^2 \times A \times C_f\right) = 2 \times \left(\frac{1}{2} \times 1.205 \times 8^2 \times 9 \times 4.61 \cdot 10^{-3}\right) N = 2 \times (1.6)\, N = 3.2\, N$$

The flow is most likely transitional, therefore to define a boundary layer thickness is not of much use.

5.(20%) Consider a converging/diverging nozzle where the flow is choked. The area ratio is

$$\frac{A_t}{A_e} = 0.6$$

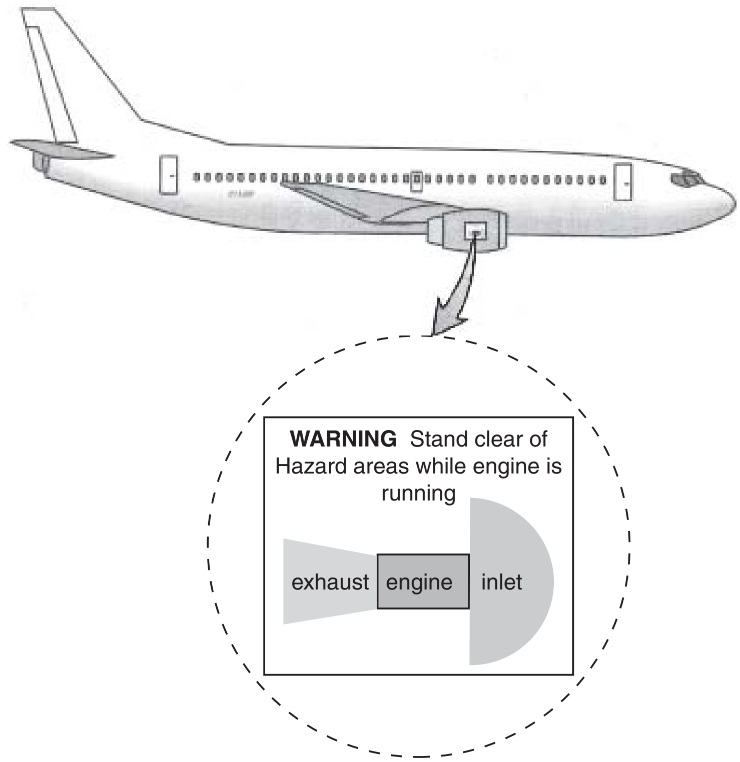
A normal shock wave sits at the exit plane. Determine the pressure ratio p_e/p_o.



$A_t/A_e = 0.6 \Longrightarrow M_1 = 2 \Longrightarrow$ for isentropic flow $p_1/p_o = 0.13$, and for the shock $M_2 = 0.58$, $p_2/p_1 = 4.5$

$$\frac{p_e}{p_o} = \frac{p_1}{p_o} \times \frac{p_2}{p_1} = 0.13 \times 4.5 = 0.58$$

6.(20%) Below is a common warning sign seen on the cowling of all jet engines. Its a warning designed to alert ground crews to areas where the influence of the engine poses a danger. Based on your understand of fluid dynamics, explain why the inlet (right) and exhaust (left) warning areas are those shapes. (10+10)



INLET: Potential flow, sink flow, sucks in from all directions when not moving, no separation.

EXHAUST: Separated flow, well directed jet, with little influence around except for a small entrainment velocity.