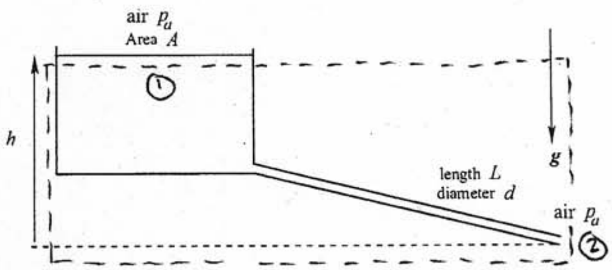
# of Exams: 139
Mean: 148.7
Standard Deviation: 26.3

UNIVERSITY OF CALIFORNIA, BERKELEY
MECHANICAL ENGINEERING
ME106 Fluid Mechanics
2nd Test, S08 *Prof S. Morris*

NAME SOLUTIONS

1.(60) The large reservoir drains through a long cylindrical pipe in which the power loss is given by $\frac{1}{2}\dot{m} f V^2 \frac{L}{d}$. Derive the differential equation giving dh/dt in terms of h, the friction factor f, and the constants shown in the figure. (You are not asked to solve the differential equation.)

Question 1
Mean: 46.5 / 60
Standard Dev: 14.2



+10: Stating that you will use a Balance of Mech. En.

Balancing <u>mechanical</u> energy on the control volume shown, we obtain

$$\dot{m}\left[\frac{1}{2}V^2 + \frac{\cancel{p}}{\rho} + gz\right]_1^2 = \text{Shaft power} - \text{power loss}$$

$p = p_a$ $\qquad$ Shaft power $= 0$ $\qquad$ $-\frac{1}{2}\dot{m} f V^2 \frac{L}{d}$

+15: Arriving at this expression or equivalent

$$\Rightarrow \quad \frac{1}{2}V^2\left(1 + f\frac{L}{d}\right) - gh = 0 \quad \text{i.e.} \quad V = \left[2gh \Big/ \left(1 + f\frac{L}{d}\right)\right]^{1/2}$$

+10: Stating that you will use a Balance of Mass

Balancing mass on same CV requires that $A\frac{dh}{dt} + V\frac{\pi d^2}{4} = 0$ (net mass out flow = 0)

+15: Arriving at this expression
Note: dh/dt = -v1

$$\Rightarrow \quad \boxed{\frac{dh}{dt} + \frac{\pi d^2}{4A}\left(\frac{2gh}{1 + f\frac{L}{d}}\right)^{1/2} = 0}$$

+10: Correct Final Result

SOLN

IN BLOCK LETTERS PRINT YOUR NAME ON THIS PAGE

-5 : Any sign errors in you expressions or math AND/OR if you incorrectly concluded dh/dt was positive.
-5 : Your final solution contained terms not in the list of knowns, i.e. A1, A2, v

Full credit if you elected to leave v1 = -dh/dt in your energy expression rather than taking it to be neglible AND correctrly worked out the remainder of the solution.

Question 2
Mean: 49.5 / 60
Standard Dev: 16.4

2. (60) An aircraft cruises subsonically at an elevation where the atmospheric temperature and pressure are respectively T_a and p_a. Assuming the Bernoulli equation in either of the two forms given in the lecture notes, and a suitable isentropic relation, *derive* an expression giving the speed V of the aircraft in terms of p_a, T_a, the measured stagnation pressure p_0 and the constants γ and c_p. (You will not receive credit for simply writing down the answer.)

BE along stagnation SL from ∞ to stagnation point:

$$\frac{1}{2}V^2 + c_p T_a = c_p T_0 \qquad (a)$$

+15: Correctly stating Compressible Bernoulli in this form or the alternate form presented in the reader.

But $\quad T/T_a = (p/p_a)^{1-1/\gamma}$

$$\Rightarrow \qquad T_0 = T_a (p_0/p_a)^{1-1/\gamma} \qquad (b)$$

+15: Correctly stating an Isentropic Relation

+20 : Correct and proper math and algebra to get from (a) and (b) to the solution.

-5: Small errors such as losing a 2
-10: Mathematical errors

Eliminating T_0 between (a), (b) we obtain

$$\boxed{V = \sqrt{2 c_p T_a}\left\{\left(\frac{p_0}{p_a}\right)^{1-1/\gamma} - 1\right\}^{1/2}}$$

+10: Having the correct solution w/o any errors

SOL'N

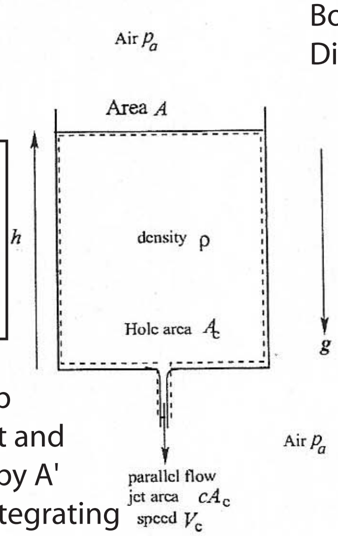
Question 3
Mean: 52.6 / 80
Standard Dev: 20

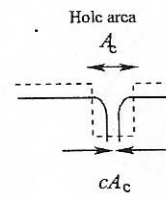
3. **(80)** The large tank is draining through a small hole of area A_e. The smaller figure shows the detail near the exit hole. Specifically, below the exit, the streamlines contract to form a jet with area cA_e, where $c < 1$ is the contraction coefficient; the speed V_e within that jet is given by the Torricelli theorem as $V_e = \sqrt{2gh}$. By balancing vertical momentum on the control volume shown, show that

$$c = \tfrac{1}{2} + \frac{1}{2\rho g h A_e}\int_{A'} (\rho g h + p_a - p)\,\mathrm{d}A. \qquad (A)$$

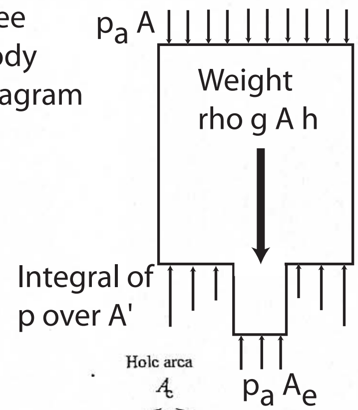
The integral is calculated over the area $A' = A - A_e$ of the tank bottom, excluding the exit hole; the liquid pressure on that area is p.

Hints. (a) The liquid weight is significant. (b) At the upper surface, the liquid has negligible momentum. (c) To express the result of the momentum balance in the form (A), you may find it useful at the end to note that $\rho g h A = \rho g h A_e + \rho g h(A - A_e)$.





Free Body Diagram



① Resultant vertical force (downwards) on contents of CV

+10: Each of the forces below (4 total)

$$= p_a A + \rho g h A - p_a A_e - \int_{A'} p\,dA$$

top weight exit bottom

If using gauge pressure: +10: Weight
+30: p-pa integrated over A'

$$= p_a(A - A_e) + \rho g A h - \int_{A'} p\,dA$$

-5: Treating p as a constant and multiplying by A' instead of integrating

$$= \rho g A h + \int_{A'} (p_a - p)\,dA$$

$$= \rho g h A' + \rho g h A_e + \int_{A'} (p_a - p)\,dA \qquad \text{(by HINT (c))}$$

$$= \rho g h A_e + \int_{A'} (\rho g h + p_a - p)\,dA \qquad ①$$

② Net flowrate vertical momentum (downwards) out of CV

$$= \rho (A_e c) V_e^2$$

Jet area

+15: This expression. -10: Losing c term
-5: Small errors

$$= 2\rho A_e c g h \qquad ②$$

+15: Correctly plugging in for velocity to get this expression.

END

2s08–3

Equating ①, ② and solving for c we obtain

$$\boxed{c = \frac{1}{2} + \frac{1}{2\rho A_e g h}\int_{A'} (\rho g h + p_a - p)\,dA}$$

+10: Correct Final Expression

QED.