CS174 **Midterm 1** **Spring 99**
J. Canny **Mar 2**

This is a closed-book exam with 4 questions. You have 80 minutes. All questions are worth equal points, so be sure to budget 20 minutes per question. You are allowed to use the formula sheet that will be handed out with the exam. No other notes are allowed. Calculators are OK. Write all your answers in this booklet. Good luck!

NAME____

SID Number____

1. Suppose a positive integer k is chosen uniformly at random from $\{1, \ldots, 1200\}$. Then the following are random variables:

$X = 1$ if k is a multiple of 4, 0 otherwise
$Y = 1$ if k is a multiple of 5, 0 otherwise
$Z = 1$ if k is a multiple of 6, 0 otherwise

(a) What is $E[X]$?

(b) What is $E[Y + Z]$?

(c) What is $E[XZ]$?

2. Let a biased coin have $Pr[Heads] = p$, where p is not necessarily 0.5. The coin is tossed repeatedly, until a total of 3 heads (not necessarily consecutive) have appeared. Let X be a random variable which is the number of tosses up to and including the 3rd head.

   (a) What is $E[X]$?

   (b) What is $\sigma^2[X]$?

   (c) What is the distribution of X? i.e. give $Pr[X = k]$ as a function of k.

3. Suppose we run the proposal algorithm (for stable marriages) on m males and n females. The algorithm is the same as before, namely:

Males Each unmarried male proposes to the highest-ranked female on his preference list who has not turned him down before.

Females Each female accepts a proposal if she is not married, or if the proposer ranks higher on her preference list than her current spouse.

Suppose further that we implement proposals in rounds. In each round, all the unmarried males propose to their current favorite. Each female accepts the highest ranked proposal in that round, unless she has a spouse who ranks higher. This is equivalent to the usual algorithm. Assume males and females have random preference lists.

(a) How large should m be (in terms of n) to be confident every female receives a proposal in the first round?

(b) If $m = n$, what is the expected number of unmarried males (or females) after the first round?

(c) If $m \neq n$, what is the expected number of females who receive more than one proposal in the first round? Simplify your result as much as possible, assuming large n and m.

4. The following examples define random variables and probabilities. In each case, suggest a tail bound (Markov, Chebyshev or Chernoff) that could be used to compute a bound on the probability. Choose the method that is applicable and gives the best bound. Then compute the bound. In each case, assume $\mu = E[X]$:

   (a) Let X have the uniform distribution on $\{1, \ldots, n\}$. What is $Pr[X > 1.5\mu]$?

   (b) Let X have the binomial distribution with parameters $p = 0.1$ and $n = 1000$. What is $Pr[X > 4\mu]$?

   (c) Let X have the geometric distribution with parameter $p = 0.2$. What is $Pr[X > 3\mu]$?