CS 174 Spring 1996
Midterm Exam 2
Professor M. Blum

Problem #1

MUFFLER MANIA (OR, TURNING 2-SIDED COINS INTO 3-SIDED COINS)
Allison, Barbara, and Cindy are walking in the field one day when they find a muffler. They decide that it most likely does not belong to anyone and that one of them should keep it. They need a way to decide who should keep the muffler. Each woman should have a probability of 1/3 of keeping the muffler. They only have a fair coin as a tool.

Allison proposes they toss the coin twice.
If it comes up HH, Allison wins.
If it comes up HT, Barbara wins.
If it comes up TH, Cindy wins.
If it comes up TT, it's a draw and they try again, repeatedly, until they get a winner.

a) Under Allison's scheme, what is the expected number of coin tosses?

Barbara has another scheme. She proposes using the coin to generate the digits of a rational number z between 0 and 1 written in base 2. For example, the coin tosses HTHTHTHHH would correspond to the number z = .101010111 .

Barbara recalls that the binary equvilants of 1/3 and 2/3 in base 2 are
1/3 = .0101010101010101010101...
2/3 = .1010101010101010101010...
(Note that 1/3 + 2/3 = .11111111111... = 1 .)

If $0 < z < 1/3$ then Allison wins.
If $1/3 < z < 2/3$ then Barbara wins.
If $2/3 < z < 1$ then Cindy wins.

b) What is the expected number of coin tosses under Barbara's scheme?
c) Which scheme requires the fewest expected number of coin tosses?

Problem #2

POCKET CHANGE
Consider tossing a coin that has probability $p = 1/20$ of Heads.
a) If you flip it $k = 5$ times, how many Heads do you expect?
(Use the definition of expectation.)
b) What is the probability that you get one or more Heads from the $k = 5$ coin flips?
c) Explain why it is that, for every probability p and positive integer k, the answer to part a) is always an upper-bound on the answer to part b). Or is it?

Problem #3

WALKING WITH CONFIDENCE
In class, we considered random walks on undirected graphs. We found that in a graph with m edges and n nodes, the expected time to visit all nodes was $2m(n-1)$.

a) After how many steps can you be 50% sure you visited all nodes? "50% sure" means that for the given graph, at least 50% of the walks (of that length) visit all nodes.

b) Repeat for 75%, 95%, and 100%.

Problem #4

YOU BET YOUR LIFE (COINS OR DICE?)
Connie tosses a fair coin n times. She wins if she gets a Head n times in a row.
Dick tosses an n-sided die n times. He wins if the n outcomes are all different.
Who has the best chance to win?

Answer the question for:
a) $n = 2$.
b) $n = 6$.
c) $n = oo$.

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