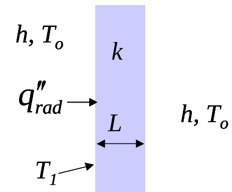
# ME109 – Heat Transfer
## Midterm 1- Fall'00
## Instructor: Prof. A. Majumdar
Oct. 18, 2000; 10:10 am - 11:30 am; Maximum Points = 40

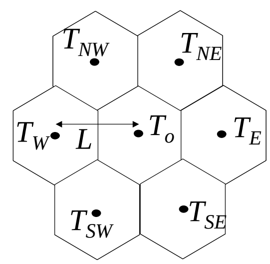
NOTE: This is an open book, open notes exam.

1. (a) In the Fourier law of heat conduction, the heat flux is written as $q'' = -k\nabla T$, where $\nabla T$ is the temperature gradient and $k$ is the thermal conductivity of the medium. What is the purpose of the negative sign on the right hand side? What law does it help to satisfy? (3)

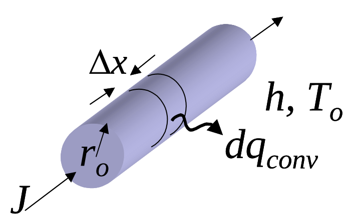
   (b) A wall of a house is irradiated by sun light with a flux of $q''_{rad}$ which is fully absorbed at the wall surface. Part of this heat flux conducts through the wall of thickness $L$ and conductivity, $k$, and is transferred by convection to $T_o$ with a convective coefficient $h$. The other part of the flux is transferred directly to the fluid at temperature $T_o$ with a convective coefficient $h$. Show that the temperature, $T_1$, of the wall surface that is irradiated by sun light is given as $T_1 = T_o + \frac{q''_{rad}}{h}\left(\frac{Bi+1}{Bi+2}\right)$, where $Bi = hL/k$. (7)




   (c) You have been asked to solve a 2-dimensional steady state heat conduction problem using numerical methods. Instead of a square mesh that one normally adopts, you decided to choose a hexagonal mesh as shown in the figure. Consider an internal node, $T_o$, as shown in the figure on the right. If the distance between the nodes is $L$, determine using energy balance, the algebraic equation for the nodal temperature $T_o$ in terms of the all the surrounding nodal temperatures and for a heat generation rate of $\dot{q}$ per unit volume. (10)




2. As an engineer in a utility company, you are asked to evaluate the performance of an electrical fuse. Consider a very long cylindrical wire of radius, $r_o$ [m], made of a material of density, $\rho$ [kg/m$^3$], heat capacity, $C$ [J/kg-K], thermal conductivity, $k$ [W/m-K], and electrical resistivity, $\beta$ [$\Omega$-m]. The wire is suspended in a fluid at temperature, $T_o$, such that the heat transfer coefficient is $h$ [W/m$^2$-K]. Initially, the wire is at temperature $T_o$, which is the fluid temperature. At $t = 0$, a current starts to flow such that the current density in the wire is $J$ [Amps/m$^2$] and wire temperature starts to increase due to heat generation by Joule heating. Assume there are no temperature gradients along the length of the wire and also the following values: $h$ =100 W/m$^2$-K, $k$ = 200 W/m-K, $r_o$ = 0.002 m, $\rho$ =2000 kg/m$^3$, $C$ = 500 J/kg-K, $T_o$ = 300 K, and $\beta = 10^{-6}$ $\Omega$-m




   (i) Based on first law of thermodynamics, develop a governing equation for the time evolution of the wire temperature. Note that for a length, $\Delta x$, of the wire, the electrical resistance of the wire is $R = \beta\Delta x/A$ and the Joule heating rate over $\Delta x$ is $I^2R$ or $(JA)^2R$ where $A$ is the cross-sectional area of the cylindrical wire (7).

   (ii) Solve the governing equation to determine the time evolution of the wire temperature (7).

   (iii) What is the minimum current density, $J_{min}$, for which the wire will reach its melting point of 2000 K? If $J = 2J_{min}$, how long will it take for the wire to reach the melting point? (6) (20)