CS174

Spring 98

Feb24

J. Canny

MIDTERM I

This is a closed book exam with 5 questions. You are allowed to use the formula sheet that will be handed out with the exam. No other notes are allowed. Calculators are OK. Write all your answers in this booklet. The score for each question is shown alongside the question. Make sure you attempt all of them. Good Luck!

•@

Name: _____

•@

•@

SID Number: _____

•@

•@

•@

•@

1.(20 points) Let a biased coin have Pr [Heads] = p , where p is not necessarily 0.5. This coin can still be used to simulate fair coin tosses by tossing it twice, where if the actual sequence of tosses is HT, the output is "H", while if the actual sequence of tosses is TH, the output is "T". If the actual tosses are both the same, repeat the process. Let an "event" be a pair of tosses, and let the random variable X count the number of events until "h" or "T" is output, including the last, so X >= 1.

a. (5 points) What kind of distribution does X have (give a name)?
b. (5 points) What is E[X] is terms of p?
c. (10 points) How could you use the same coin to simulate a fair 3-sided dice, i.e. to output 1,2, or 3 with equal probability?

•@

•@

•@

2.)(20 points) Let X be a random variable with E[X] = 3, Var [X] = 4, and let Y be a random variable with E[Y] = 5.

a. (5 points) What is E[3X]?
b. (5 points) What is E[X + Y]?
c. (5 points) What is E[XY]?
d. (5 points) What is E[sqr(X)]?

•@

•@

•@

3.)(20 points) Let p and q be two probability vectors, that is, vectors of non-negative real numbers whose sum is 1. Let M as shown below be a payoff matrix for a two-player zero-sum game.

M =

2 1 0

2 2 -2

-2 -1 0

•@

If q = (0.3, 0.3, 0.4), what is the p that maximizes p^T Mq?

•@

•@

•@

4.)(20 points) Pizza Palace has implemented a new electronic dispatching system but is having trouble with it. Because of that, m pizzas are dispatched independently and uniformly at random to n customers.

a. (10 points) How large should m be (in terms of n) to be confident that everyone gets a pizza? You don_t need to give an exact bound, but rather a value that is accurate as n -> infinity.
b. (10 points) How large should m be to be confident that no one gets more than one pizza? You don_t need to give an exact bound, but rather a value that is accurate as n -> infinity.

•@

•@

•@

5.)(20 points) Let a fair coin be tossed 16 times. Let X be the number of heads, then E[X] = 8 and Var[X] = 4.

a. (5 points) Give a Markov bound for Pr[X >= 12].
b. (5 points) Give a Chebyshev bound for Pr[X >= 12].
c. (10 points) Give a Chernoff bound for Pr[X >= 12].

•@

•@

•@

THE END!!!