

Figure 1: The histogram of scores. Vital statistics: $N = 147$, $\mu = 43$, $\sigma = 14$.

1 Problem 1

See next page

2 Problem 2

The five parts were worth 2 points each. The answers are as follows: (a) Bayesian; (b) Decision tree; (c) Multilayer perceptron; (d) k -NN; (e) Single layer perceptron.

3 Problem 3

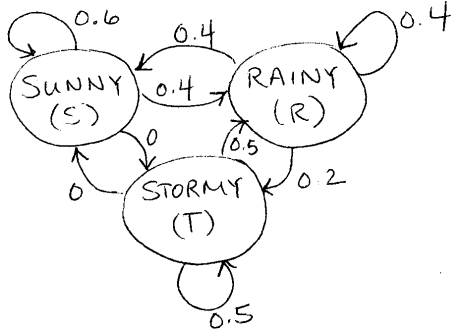
The three parts were worth 5 points each. Let w_x and w_y denote the weight of the edges from inputs x and y to the output node, and let t denote the threshold of the output node (so the activation function is $\text{step}_t(z) = 1$ if $z \geq t$ and 0 otherwise, as on page 569). Note that using a threshold of t has the same effect as using a threshold of 0 but adding a bias node with input clamped to 1 and with weight $-t$.

(a) NOR. $w_x = -2$, $w_y = -2$, and $t = -1$ works.

(b) EQUAL. Not possible, since the inputs for which EQUAL=0 are not linearly separable from the inputs for which EQUAL=1. (Notice also that EQUAL = NOT XOR.)

(c) ONE. $w_x = 0$, $w_y = 0$, and $t = -1$ works.

1. (a) Stochastic automaton



$$(b) \alpha(q_1) = \begin{bmatrix} 5/12 \\ 5/12 \\ 2/12 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0.4 \\ 0 \end{bmatrix} = \begin{bmatrix} 5/12 \\ 2/12 \\ 0 \end{bmatrix}$$

$$(c) \alpha(q_2) = \sum_{q_1} \alpha(q_1) a_{q_1 \rightarrow q_2} P(u_2 | q_2)$$

$$= \begin{pmatrix} [5/12 \ 2/12 \ 0] \begin{bmatrix} 0.6 & 0.4 & 0 \\ 0.4 & 0.4 & 0.2 \\ 0 & 0.5 & 0.5 \end{bmatrix} \end{pmatrix}^T \cdot \begin{bmatrix} 0 \\ 0.6 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 7/50 \\ 1/30 \end{bmatrix}$$

$$(d) P(q_2=R | u_1, u_2) = \frac{\alpha(q_2=R) \beta(q_2=R)}{\sum_{q_2} \alpha(q_2) \beta(q_2)} ; \text{ AND } \beta(q_2) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{(7/50)(1)}{[(0)(1) + (7/50)(1) + (1/30)(1)]} = 0.808 \Rightarrow \sim 81\%$$

USEFUL INFO

vectors: $\begin{bmatrix} S \\ R \\ T \end{bmatrix}$

Priors:

$$P(S) = 5/12$$

$$P(R) = 5/12$$

$$P(T) = 2/12$$

Emissions:

	u	$\neg u$
S	0	1
R	0.6	0.4
T	1	0

4 Problem 4

- a). The joint for Bayes net A is: $\mathbf{P}(X, Y, Z) = \mathbf{P}(Z|Y)\mathbf{P}(Y|X)\mathbf{P}(X)$. Therefore, $\mathbf{P}(X, \neg Y, \neg Z) = \mathbf{P}(\neg Z|\neg Y) * \mathbf{P}(\neg Y|X) * \mathbf{P}(X) = .5 * .8 * .6 = 0.24$
- b). To completely specify Bayes net B, we need the prior probability for node Y and the conditional probability tables (CPTs) for nodes X and Z . So, we want $\mathbf{P}(Y)$, $\mathbf{P}(X|Y)$, and $\mathbf{P}(Z|Y)$ (notice that we are already given $\mathbf{P}(Z|Y)$ in the problem statement). Thus:

$$\mathbf{P}(Y) = \mathbf{P}(Y|X)\mathbf{P}(X) + \mathbf{P}(Y|\neg X)\mathbf{P}(\neg X) = .2 * .6 + .7 * .4 = .12 + .28 = 0.4$$

$$\mathbf{P}(X|Y) = \frac{\mathbf{P}(Y|X)\mathbf{P}(X)}{\mathbf{P}(Y)} = \frac{.2 * .6}{.4} = 0.3$$

$$\mathbf{P}(X|\neg Y) = \frac{\mathbf{P}(\neg Y|X)\mathbf{P}(X)}{\mathbf{P}(\neg Y)} = \frac{(1 - .2) * .6}{1 - .4} = 0.8$$

