1. Measured with respect to the horizontal, if the angular position of link *CB* is θ, the angular position of crank *OA* is 2θ. Hence,

$$\omega_{BC} = \frac{1}{2}\omega_{AO} = 5\,\text{rad/s}$$

Attach a rotating *x*-*y* frame to *C* fixed on link *CB* with the *x*-axis along *CB*. Then

$$\begin{aligned}\mathbf{a}_A &= \mathbf{a}_C + \dot{\boldsymbol{\omega}}\times\mathbf{r} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}} \\ &= \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + 2\boldsymbol{\omega}\times\mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}} \\ &= 5\mathbf{k}\times(5\mathbf{k}\times 0.4\cos 30^\circ\mathbf{i}) + 10\mathbf{k}\times v_{\text{rel}}\mathbf{i} + a_{\text{rel}}\mathbf{i} \qquad (1)\end{aligned}$$

The pin *A* can only slide along the slot on link *CB*, therefore $\mathbf{v}_{\text{rel}}$ and $\mathbf{a}_{\text{rel}}$ act along the link *CB*. On the other hand, since *OA* rotates with a constant angular velocity of 10 rad/s,

$$\mathbf{a}_A = (\mathbf{a}_{A/O})_n = -0.2(10)^2\cos 30^\circ\mathbf{i} - 0.2(10)^2\sin 30^\circ\mathbf{j} \qquad (2)$$

It follows from Eqs. (1) and (2) that

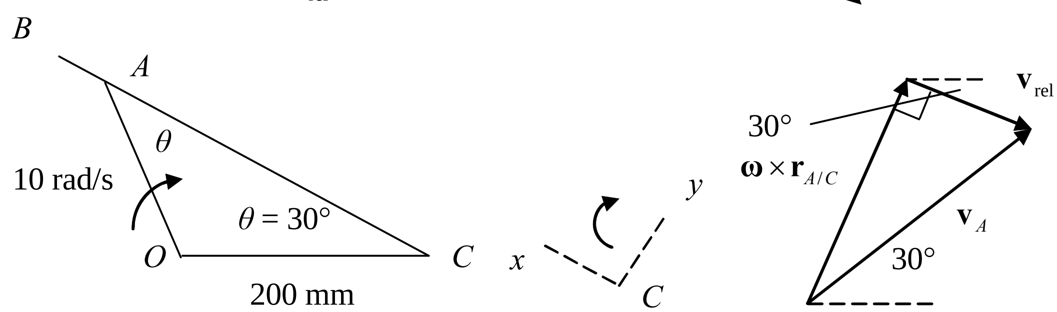
$$-20\cos 30^\circ\mathbf{i} - 20\sin 30^\circ\mathbf{j} = -10\cos 30^\circ\mathbf{i} + 10\mathbf{k}\times v_{\text{rel}}\mathbf{i} + a_{\text{rel}}\mathbf{i}$$

Equating **i** coefficients,

$$a_{\text{rel}} = -10\cos 30^\circ = -8.66\,\text{m/s}$$

As a byproduct, equating **j** coefficients,

$$v_{\text{rel}} = -2\sin 30^\circ = -1\,\text{m/s}$$



2. For the square panel,

$$\sum F_x = m(a_G)_x \quad\Rightarrow\quad T\cos 45^\circ = m(a_G)_x \qquad (1)$$

$$\sum F_y = m(a_G)_y \quad\Rightarrow\quad T\cos 45^\circ - mg = m(a_G)_y \qquad (2)$$

$$\sum M_G = I_G\alpha \quad\Rightarrow\quad T\frac{b}{\sqrt{2}} = \frac{1}{6}mb^2\alpha \qquad (3)$$

There are four unknowns *T*, $(a_G)_x$, $(a_G)_y$, and α in three equations. An additional equation is provided by kinematics. For the two points *A* and *G*,

$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A}$$

Since the panel is stationary when the wire at *B* breaks, $\omega = 0$ and $v_A = 0$. Hence,

$$(a_{G/A})_n = \frac{b}{\sqrt{2}}\omega^2 = 0 \quad\Rightarrow\quad a_{G/A} = (a_{G/A})_t = \frac{b}{\sqrt{2}}\alpha$$

$$(a_A)_n = \frac{v_A^2}{b} = 0 \quad\Rightarrow\quad a_A = (a_A)_t$$

As a result, $\mathbf{a}_{G/A}$ is perpendicular to *AG*, $\mathbf{a}_A$ is along *AG*, and

$$(a_G)_x = (a_A)_x + (a_{G/A})_x = a_A \cos 45° - \frac{b}{\sqrt{2}} \alpha \cos 45°$$

$$(a_G)_y = (a_A)_y + (a_{G/A})_y = -a_A \cos 45° - \frac{b}{\sqrt{2}} \alpha \cos 45°$$

Add the above two equations to obtain

$$(a_G)_x + (a_G)_y = -\sqrt{2} b \alpha \cos 45° \tag{4}$$

Solve Eqs. (1) – (4) simultaneously,

$$T = \frac{1}{4\sqrt{2}} mg = 20.8\,\text{N}$$

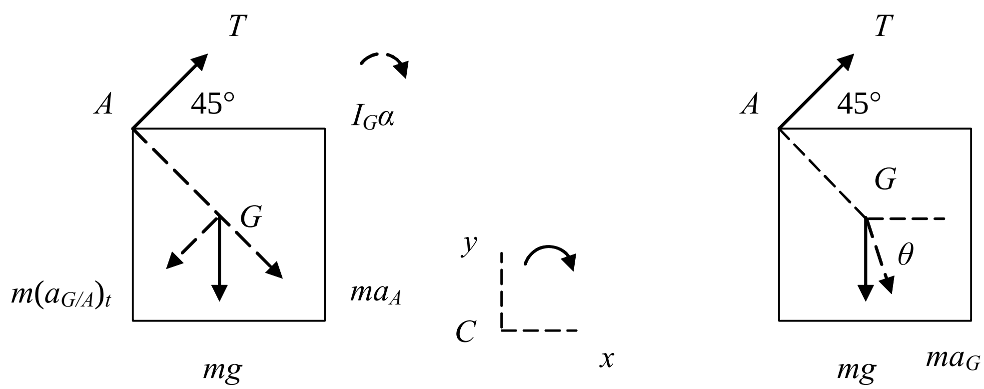
In addition, $(a_G)_x = \frac{1}{8} g$, $(a_G)_y = -\frac{7}{8} g$. Since

$$a_G = [(a_G)_x^2 + (a_G)_y^2]^2 = \frac{5\sqrt{2}}{8} g = 1.08\,\text{m/s}^2$$

$$\tan\theta = -\frac{(a_G)_y}{(a_G)_x} = 7 \quad \Rightarrow \quad \theta = 81.87°$$

$\mathbf{a}_G$ has magnitude $(5\sqrt{2}/8)g$ and makes an angle $\theta = 81.87°$ with the horizontal. Note that

$$T_{st} = \frac{mg}{2\cos 45°} = \frac{1}{\sqrt{2}} mg > T = \frac{1}{4\sqrt{2}} mg$$



3. (a) The acceleration of 4*m* can also be found by Newtonian method. Let T_1 be the tension in the upper string and T_2 tension in the lower string. Force balance on each mass gives

$$4mg - T_1 = 4m\ddot{q}_1 \tag{1}$$

$$mg + 2T_2 - T_1 = m\ddot{q}_2 \tag{2}$$

$$2mg - T_2 = 2m\ddot{q}_3 \tag{3}$$

$$mg - T_2 = m\ddot{q}_4 \tag{4}$$

There are six unknowns $\ddot{q}_1$, $\ddot{q}_2$, $\ddot{q}_3$, $\ddot{q}_4$, T_1, and T_2 in four equations. However,

$$\ddot{q}_1 = -\ddot{q}_2 \tag{5}$$

$$\ddot{q}_3 + \ddot{q}_4 - 2\ddot{q}_2 = 0 \tag{6}$$

Combine Eqs. (1) and (2) to obtain

$$3mg - 2T_2 = 4m\ddot{q}_1 - m\ddot{q}_2 = 5m\ddot{q}_1$$

Combine Eqs. (3) and (4) to get

$$4mg - 3T_2 = 2m(\ddot{q}_3 + \ddot{q}_4) = -4m\ddot{q}_1$$

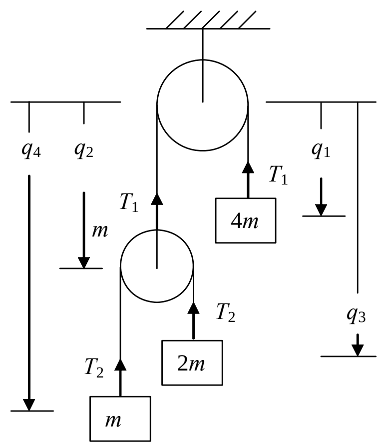
Upon solution,

$$\ddot{q}_1 = \frac{1}{23}g \qquad \downarrow$$

(b) Due to motion of the masses 2*m* and *m* on the lower pulley, forces on each side of the upper pulley do not balance even though the total mass on each side is the same. For example, it can be readily shown that

$$T_1 - 2T_2 - mg = \frac{1}{23}mg$$

Thus the mass 4*m* has a downward acceleration.



4. (a) Let the rope break in position $\theta = \alpha$. Before the rope breaks, the bag travels in a circle of radius *l*. In any position $\theta \le \alpha$,

$$\sum F_t = ma_t \quad \Rightarrow \quad mg\cos\theta = ma_t$$

$$\Rightarrow \quad \dot{v} = g\cos\theta \tag{1}$$

$$\sum F_n = ma_n \quad \Rightarrow \quad T - mg\sin\theta = m\frac{v^2}{l} \tag{2}$$

There are four variables *T*, θ, *v*, and $\dot{v}$ in two equations. From kinematics and Eq. (1),

$$\dot{v} = \frac{dv}{d\theta}\dot{\theta} = \frac{dv}{d\theta}\frac{l\dot{\theta}}{l} = \frac{vdv}{ld\theta} = g\cos\theta \quad \Rightarrow \quad \int_0^v vdv = \int_0^\theta g\cos\theta\, ld\theta$$

$$\Rightarrow \quad v^2 = 2gl\sin\theta \tag{3}$$

Substitute Eq. (3) into (2),

$$T - mg\sin\theta = m\frac{2gl\sin\theta}{l} \tag{4}$$

When $\theta = \alpha$, $T = 2mg$. It follows from Eq. (4) that

$$2mg - mg\sin\alpha = m\frac{2gl\sin\alpha}{l} \quad \Rightarrow \quad \alpha = \sin^{-1}\frac{2}{3} = 41.81°$$

(b) Set up a rectangular system at *A*. When the rope breaks, the position of the bag is

$$(x_0, y_0) = (l - l\cos\alpha, -l\sin\alpha) = (2.546, -6.667)$$

In that position, $v = \sqrt{2gl\sin\alpha} = 11.431$ and the vertical distance to fall before reaching the level C is $h - |y_0| = h - 6.667 = 23.333$. Along the y-direction,

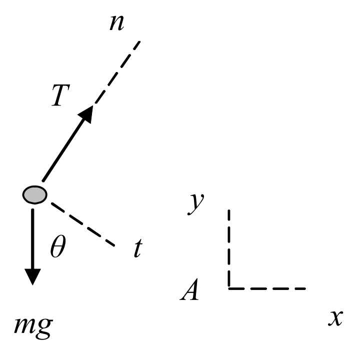
$$y = v_y t - \frac{1}{2}gt^2 \qquad \Rightarrow \qquad -23.333 = (-11.431\cos\alpha)t - \frac{1}{2}gt^2$$

$$\Rightarrow \qquad t = 1.479$$

Along the x-direction, the horizontal distance traveled in time t is

$$x = v_x t = 11.431\sin\alpha(1.479) = 11.271$$

Thus the horizontal distance of C from A is $11.27 + x_0 = 11.27 + 2.55 = 13.82\,\text{m}$



5. Let m be the mass of square frame. The moment of inertia about the mass center G is

$$I_G = 4\left(\frac{1}{12}\frac{m}{4}b^2 + \frac{m}{4}\left(\frac{b}{2}\right)^2\right) = \frac{1}{3}mb^2$$

By the parallel-axis theorem,

$$I_B = I_G + m\left(\frac{b\sqrt{2}}{2}\right)^2 = \frac{5}{6}mb^2$$

(a) After A has dropped a distance b, G has dropped a distance $b/2$, and line AB assumes the new position $A'B'$. To show that B' is an instantaneous center of zero velocity, attach an x-y system to A'. Let θ be the angle between line AB and the horizontal. As $B \to B'$, the coordinates of B are

$$(x, y) = (\sqrt{2}b\cos\theta, 0)$$

$$\Rightarrow \qquad (\dot{x}, \dot{y}) = (\sqrt{2}b\sin\theta\,\dot{\theta}, 0) \to (0, 0)$$

as $\theta \to 0$. Between the initial position with $\theta = 45°$ and the final position with $\theta = 0°$,

$$\Delta T + \Delta V_g = 0$$

$$\Rightarrow \qquad \frac{1}{2}I_{B'}\omega^2 = \frac{1}{2}\left(\frac{5}{6}mb^2\right)\omega^2 = mg\frac{b}{2}$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{6g}{5b}}$$

$$\Rightarrow \qquad v_A = \sqrt{2}b\omega = \sqrt{\frac{12gb}{5}} \qquad \downarrow$$

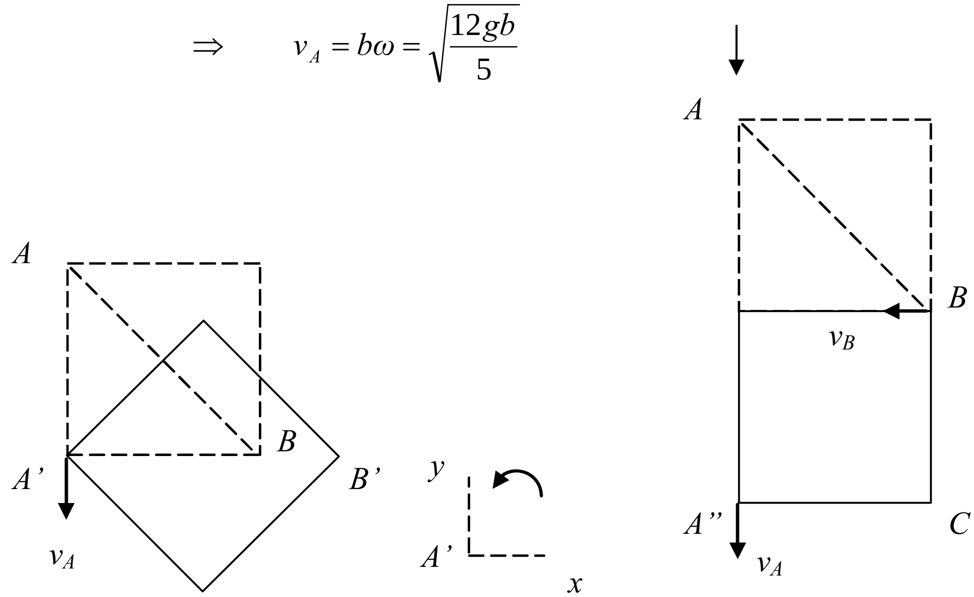
(b) After A has dropped a distance $2b$, G has dropped a distance b, and line AB assumes the position $A''B$. The instantaneous center of zero velocity in this position is *C*. Between the initial position with $\theta = 45°$ and the final position with $\theta = -45°$,

$$\Delta T + \Delta V_g = 0$$

$$\Rightarrow \qquad \frac{1}{2} I_C \omega^2 = \frac{1}{2}\left(\frac{5}{6} m b^2\right)\omega^2 = mgb$$

$$\Rightarrow \qquad \omega = \sqrt{\frac{12g}{5b}}$$

$$\Rightarrow \qquad v_A = b\omega = \sqrt{\frac{12gb}{5}}$$



(a) (a) (b)