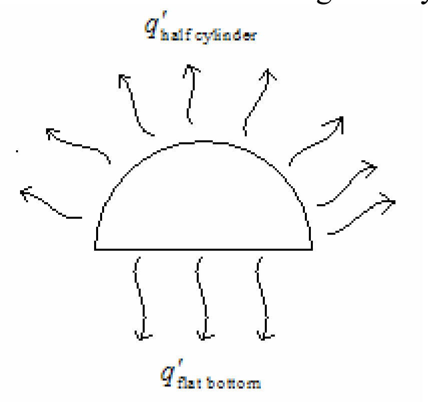
1. (45)

a) The heat transfer can be gotten by performing an energy balance around a slice of the object



$$q'_{total\_out} = q'_{\text{half cylinder}} + q'_{\text{flat bottom}} = h_{cylinder}\pi R\left(T_{surf} - T_{air}\right) + h_{plate}2R\left(T_{surf} - T_{air}\right)$$

Only the heat transfer coefficients are unknown, so the problem is to calculate these.

Properties: at T$_{\text{film}}$ = 400K

$$\nu_{air} = 26.41\times10^{-6}\,\frac{m^2}{s}$$

$$k_{air} = 33.8\times10^{-3}\,\frac{W}{m\bullet K}$$

$$\Pr_{air} = 0.69$$

Bottom flat plate

Calculate the Reynolds number

$$\text{Re}_L = \frac{u2R}{\nu} = \frac{10\frac{m}{s}0.008m}{26.41\times10^{-6}\frac{m^2}{s}} = 3029$$

This is well below the transition value of 5e5, so laminar flow equations are used.

Correlation

$$\overline{Nu} = 0.664\,\text{Re}_x^{1/2}\,\Pr^{1/3} = 0.664\sqrt{3029}\left(0.69\right)^{1/3} = 32.3$$

Back out h

$$\bar{h}_{plate} = \frac{\overline{Nu}\bullet k_{air}}{2R} = \frac{31.32\bullet33.8\times10^{-3}\frac{W}{m\bullet K}}{0.008m} = 136.4\frac{W}{m^2K}$$

Top half cylinder – cylinder in a cross flow

Calculate Re

$$\mathrm{Re}_D = \frac{uD}{\nu} = \frac{10\frac{m}{s}0.008m}{26.41\times10^{-6}\frac{m^2}{s}} = 3029.156$$

Correlation

Two are possible for the instruction to evaluate properties at T_{film}

7.55b

$$\overline{Nu} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^{1/3} = 0.683(3029)^{0.466}\,0.69^{1/3} = 25.293$$

7.57

$$\overline{Nu} = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\mathrm{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282000}\right)^{5/8}\right]^{4/5} = 27.96$$

7.56 can be used, but properties should be looked up at the air temperature

$$\nu_{air} = 15.89\times10^{-6}\frac{m^2}{s}$$

$$k_{air} = 26.3\times10^{-3}\frac{W}{m\bullet K}$$

$$\mathrm{Pr}_{air} = 0.707$$

$$\mathrm{Pr}_{\text{air at }T_s} = 0.684$$

$$\mathrm{Re}_D = \frac{10\bullet0.008}{15.89\times10^{-6}} = 5034.6$$

$$\overline{Nu} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n\left(\frac{\mathrm{Pr}}{\mathrm{Pr}_s}\right)^{1/4} = 0.26(5034.6)^{0.6}(0.707)^{0.37}\left(\frac{0.707}{0.684}\right)^{0.25} = 38.37$$

Back out h

7.55b

$$\bar{h}_{cylinder} = \frac{\overline{Nu}\bullet k_{air}}{D} = \frac{25.293\bullet33.8\times10^{-3}\frac{W}{m\bullet K}}{0.008m} = 106.86\frac{W}{m^2K}$$

7.57

$$\bar{h}_{cylinder} = \frac{\overline{Nu}\bullet k_{air}}{D} = \frac{27.962\bullet33.8\times10^{-3}\frac{W}{m\bullet K}}{0.008m} = 118.14\frac{W}{m^2K}$$

7.56

$$\bar{h}_{cylinder} = \frac{\overline{Nu}\bullet k_{air}}{D} = \frac{38.37\bullet26.3\times10^{-3}\frac{W}{m\bullet K}}{0.008m} = 126.15\frac{W}{m^2K}$$

Inserting these into the energy balance

7.55b

$$q'_{total\_out} = 106.86\frac{W}{m^2K}\left(\pi 0.004m\right)\left(200K\right) + 132.33\frac{W}{m^2K}\left(0.008m\right)\left(200K\right) = 480.3\frac{W}{m}$$

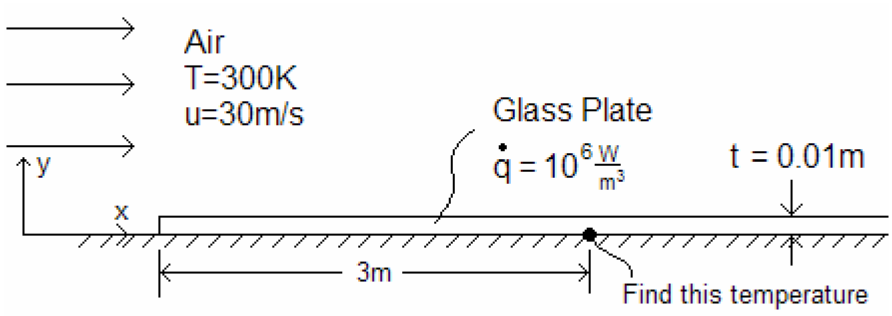
7.57

$$q'_{total\_out} = 118.14\frac{W}{m^2K}\left(\pi 0.004m\right)\left(200K\right) + 132.33\frac{W}{m^2K}\left(0.008m\right)\left(200K\right) = 508.6\frac{W}{m}$$

7.56

$$q'_{total\_out} = 126.15\frac{W}{m^2K}\left(\pi 0.004m\right)\left(200K\right) + 132.33\frac{W}{m^2K}\left(0.008m\right)\left(200K\right) = 528.8\frac{W}{m}$$

2. (45)



Set up heat transfer problem

For no conduction in the x-direction, the 1-D steady state solution for the maximum temperature is given by equation 3.43, where symmetry has been used

$$T_{\max} = \frac{\dot{q}\,t^2}{2k_{glass}} + T_{surface}$$

The unknown in this equation is the surface temperature. This can be found by applying an energy balance at the surface, matching the amount generated in a slice to that leaving through convection

$$q'' = \int_0^{y=0.01m} \dot{q}\,dy = h\left(T_{surface} - T_{air}\right)$$

Note that h is the local value.

Properties

Air at T_{film} = 350K

$$\nu_{air} = 20.92\times10^{-6}\,\frac{m^2}{s}$$

$$k_{air} = 30\times10^{-3}\,\frac{W}{m\bullet K}$$

$$\Pr_{air} = 0.7$$

Glass at 300K

$$k_{glass} = 1.4\,\frac{W}{m\bullet K}$$

Find the convection coefficient

Calculate Re

$$\mathrm{Re}_L = \frac{uL}{\nu_{air}} = \frac{30\frac{m}{s}\bullet 3m}{20.92\times10^{-6}\,\frac{m^2}{s}} = 4,302,103 \geq 10^5$$

The flow is turbulent

Correlation –

Notes

] 1) This should be a LOCAL calculation

2) The correlation should be for a constant q" boundary condition, no x conduction

$$Nu = 0.0308\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} = 0.0308\left(4302103\right)^{0.8} 0.7^{1/3} = 5544.5$$

(The constant T_s answer is $Nu = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} = 5328.5$)

Back out h

$$h = \frac{Nu \bullet k_{air}}{L} = \frac{5544.5 \bullet 30\times10^{-3}\,\frac{W}{m\bullet K}}{3m} = 55.45\frac{W}{m^2K}$$

(The constant T_s answer is $h = 53.3\frac{W}{m^2K}$)

Solve for the surface temperature

$$\int_0^{y=0.01m} \dot{q}dy = h\left(T_{surface} - T_{air}\right)$$

$$10000\frac{W}{m^2} = 55.45\frac{W}{m^2K}\left(T_{surface} - 300K\right)$$

$$T_{surface} = 480.4K$$

(The constant T_s answer is $T_s = 487.6K$)

Solve for the maximum temperature

$$T_{\max} = \frac{10^6\,\frac{W}{m^3}\left(0.01m\right)^2}{2\left(1.4\frac{W}{mK}\right)} + 480.4K = 516.1K$$

(The constant T_s answer is $T_{\max} = 523.3K$)

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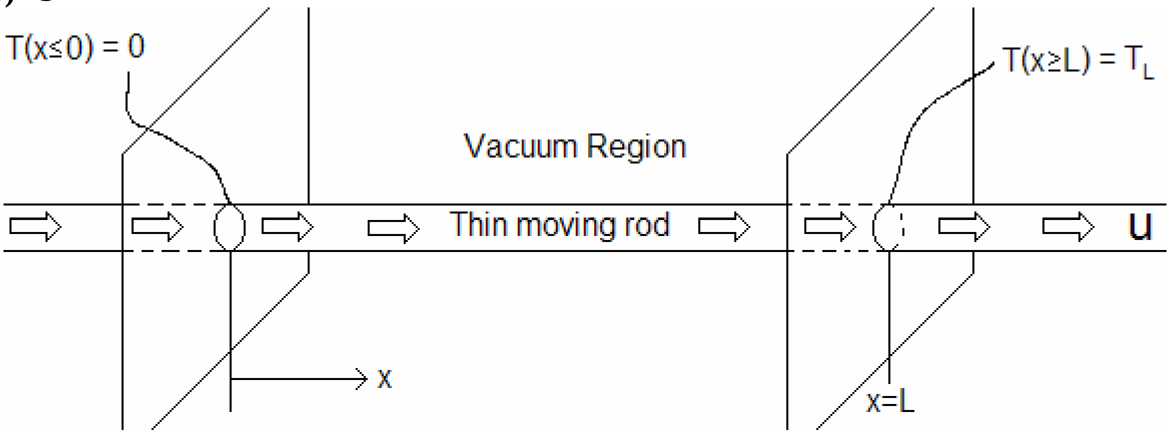
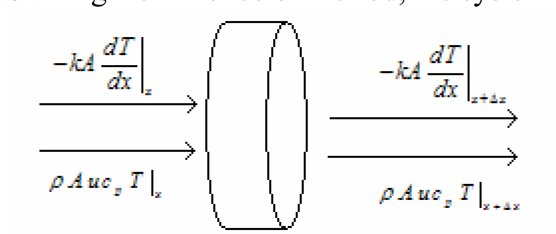


At steady state, a thin solid rod with no radial temperature variation ($T \neq T(r)$) moves at constant velocity *u* in the x-direction. The rod passes through a section of vacuum occupying $0 \leq x \leq L$. Outside this vacuum section, for $x \leq 0$ the temperature of the rod is maintained at T=0, and for $x \geq L$ the temperature of the rod is maintained at T_L. Neglecting radiation in the vacuum section, find the temperature of the rod at x=L/2.
Hint: This problem can be viewed as inviscid flow in an insulated tube

Solution

The governing differential equation can be obtained two ways:

1. Starting from a slice of the rod, the system at steady state, with no generation



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$\rho Auc_p T|_x - \rho Auc_p T|_{x+\Delta x} + \left(-kA\left.\frac{dT}{dx}\right|_x\right) - \left(-kA\left.\frac{dT}{dx}\right|_{x+\Delta x}\right) = 0$$

Divide by A, Δx

$$-\rho uc_p \frac{T|_{x+\Delta x}\ T|_x}{\Delta x} + kA\frac{\left(\left.\frac{dT}{dx}\right|_{x+\Delta x} - \left.\frac{dT}{dx}\right|_x\right)}{\Delta x} = 0$$

Take the limit as Δx goes to zero

$$\lim_{\Delta x \to 0}\left(-\rho u c_p \frac{T\big|_{x+\Delta x}\ T\big|_x}{\Delta x} + k\frac{\left(\frac{dT}{dx}\Big|_{x+\Delta x} - \frac{dT}{dx}\Big|_x \right)}{\Delta x} \right) = -\rho u c_p \frac{dT}{dx} + k\frac{d^2T}{dx^2} = 0$$

2. Starting from the general energy conservation given by equations 6.28a and 6.28b.

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu\left\{ \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + 2\left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 \right] \right\} + \dot{q}$$

This problem is 1-D with constant velocity and no generation, reducing the equation to

$$\rho c_p u \frac{dT}{dx} = k\frac{d^2T}{dx^2}$$

or equivalently,

$$\frac{d^2T}{dx^2} - \frac{u}{\alpha}\frac{dT}{dx} = 0$$

<u>Note that if any convection in the radial direction were present in this problem, this equation would have failed to capture that!</u>

This is a second order ordinary differential equation. Solving:

Convert

$$\lambda^2 - \frac{u}{\alpha}\lambda = 0$$

$$\lambda\left(\lambda - \frac{u}{\alpha} \right) = 0$$

$$\lambda = 0$$

$$\lambda = \frac{u}{\alpha}$$

=>

$$T(x) = C_1 + C_2 e^{\frac{u}{\alpha}x}$$

check

$$\frac{d^2T}{dx^2} = \frac{u^2}{\alpha^2}C_2 e^{\frac{u}{\alpha}x}$$

$$-\frac{u}{\alpha}\frac{dT}{dx} = -\frac{u^2}{\alpha^2}C_2 e^{\frac{u}{\alpha}x}$$

ok

Applying boundary conditions

1) T(x=0)=0

$$0 = C_1 + C_2$$

$$C_2 = -C_1$$

$$T(x) = C_1\left(1 - e^{\frac{u}{\alpha}x} \right)$$

2) T(x=L)=TL

$$T_L = C_1\left(1 - e^{\frac{uL}{\alpha}}\right)$$

$$C_1 = \frac{T_L}{1 - e^{\frac{uL}{\alpha}}}$$

$$T(x) = T_L\left(\frac{1 - e^{\frac{u}{\alpha}x}}{1 - e^{\frac{uL}{\alpha}}}\right)$$

The midpoint temperature is then

$$T\left(\frac{L}{2}\right) = T_L\left(\frac{1 - e^{\frac{uL}{2\alpha}}}{1 - e^{\frac{uL}{\alpha}}}\right) = T_L\left(\frac{1 - e^{\frac{\rho u c_p L}{2k}}}{1 - e^{\frac{\rho u c_p L}{k}}}\right)$$