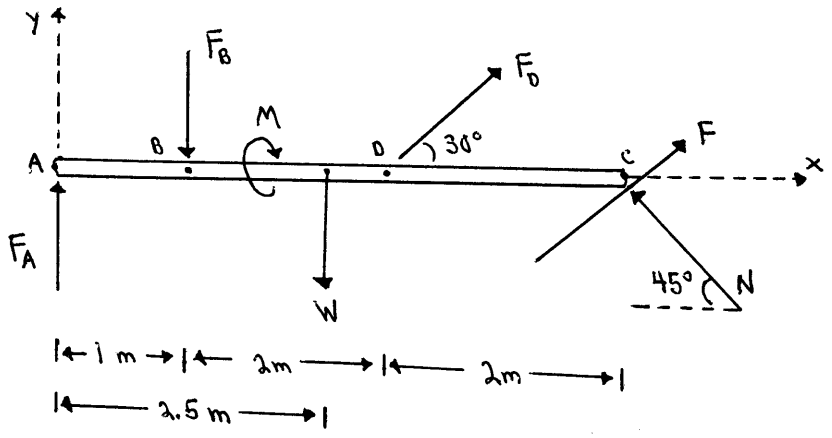
Midterm exam 1 solutions

Problem 1

a.



$M = 50\ KN\cdot m$

$F_D = 20\ KN$

$W = 10\ KN$

$\mu_s = 0.5$

b. neglect friction $\Rightarrow F = 0$

ΣF_x: $F_D\left(\frac{\sqrt{3}}{2}\right) - N\left(\frac{\sqrt{2}}{2}\right) = 0 \longrightarrow N = \sqrt{\frac{3}{2}}F_D = 10\sqrt{6}\ KN$

$\circlearrowleft +\Sigma M_A$: $-F_B(1) - M - W\left(\frac{5}{2}\right) + \left(\frac{1}{2}F_D\right)(3) + \left(\frac{\sqrt{2}}{2}N\right)(5) = 0$

$F_B = -50 - 25 + 30 + 50\sqrt{3}$

$$\boxed{F_B = 50\sqrt{3} - 45\ KN}$$

c. slip condition: $F = \mu_s N$

ΣF_x: $F_D\left(\frac{\sqrt{3}}{2}\right) - N\left(\frac{\sqrt{2}}{2}\right) + F\left(\frac{\sqrt{2}}{2}\right) = 0$

$\sqrt{3}F_D = \sqrt{2}(1-\mu_s)N = \frac{\sqrt{2}}{2}N \longrightarrow N = \sqrt{6}F_D = 20\sqrt{6}\ KN$

$\circlearrowleft +\Sigma M_A$: $-F_B(1) - M - W\left(\frac{5}{2}\right) + \left(\frac{1}{2}F_D\right)(3) + \left(\frac{\sqrt{2}}{2}N\right)(5) + \left(\frac{\sqrt{2}}{2}\mu_s N\right)(5) = 0$

$F_B = -45 + 50\sqrt{12} + 25\sqrt{12}$

$$\boxed{F_B = 150\sqrt{3} - 45\ KN}$$

Problem 1

c. alternative

if it was assumed the bar was slipping up the incline, the direction of F would be reversed in the free-body diagram (a)

under this assumption,

slip condition: $F = \mu_s N$

ΣF_x: $F_D\left(\frac{\sqrt{3}}{2}\right) - N\left(\frac{\sqrt{2}}{2}\right) - F\left(\frac{\sqrt{2}}{2}\right) = 0$

$\sqrt{3}F_D = \sqrt{2}(1+\mu_s)N = \frac{3\sqrt{2}}{2}N \longrightarrow N = \frac{\sqrt{6}}{3}F_D = \frac{20}{3}\sqrt{6}$ KN

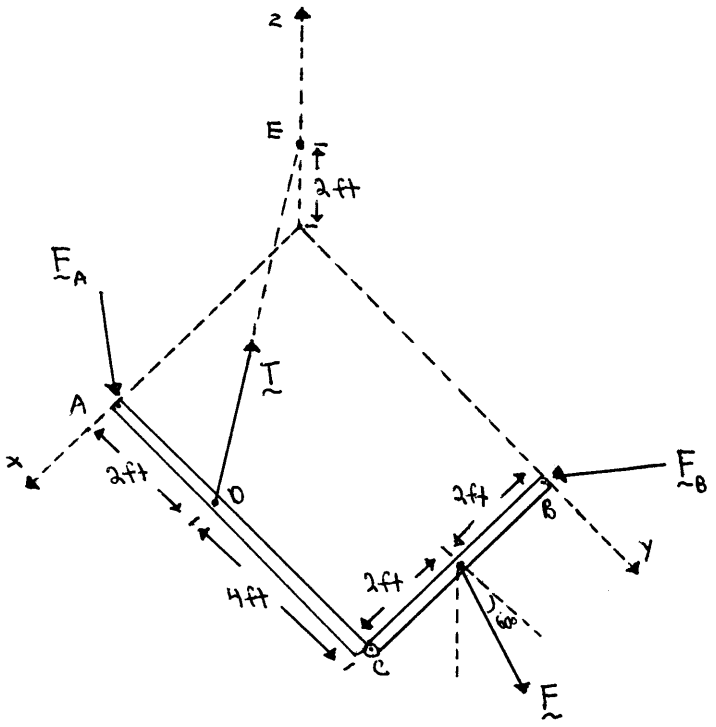
$\zeta + \Sigma M_A$: $-F_B(1) - M - W\left(\frac{5}{2}\right) + \left(\frac{1}{2}F_D\right)(3) + \left(\frac{\sqrt{2}}{2}N\right)(5) - \left(\frac{\sqrt{2}}{2}\mu_s N\right)(5) = 0$

$F_B = -45 + \frac{50}{3}\sqrt{12} - \frac{25}{3}\sqrt{12}$

$$\boxed{F_B = \frac{50}{3}\sqrt{3} - 45 \text{ KN}}$$

Problem 2

a.



$$\underset{\sim}{F} = 50\,\underset{\sim}{j} - 50\sqrt{3}\,\underset{\sim}{K}$$

$$\underset{\sim}{F}_A = F_{Ax}\,\underset{\sim}{i} + F_{Ay}\,\underset{\sim}{j} + F_{Az}\,\underset{\sim}{K}$$

$$\underset{\sim}{F}_B = F_{Bx}\,\underset{\sim}{i} + F_{By}\,\underset{\sim}{j} + F_{Bz}\,\underset{\sim}{K}$$

$$\underset{\sim}{T} = T\,\underset{\sim}{u}_{DE}$$

b.

$$\underset{\sim}{T} = T\,\underset{\sim}{u}_{DE} = T\,\frac{\underset{\sim}{r}_{DE}}{r_{DE}} \quad \text{where} \quad \underset{\sim}{r}_{DE} = \underset{\sim}{r}_E - \underset{\sim}{r}_D = 2\underset{\sim}{K} - (4\underset{\sim}{i} + 2\underset{\sim}{j})$$

$$r_{DE} = \sqrt{2^2 + (-4)^2 + (-2)^2} = 2\sqrt{6}$$

$$\longrightarrow \quad \underset{\sim}{T} = \frac{T}{\sqrt{6}}\left(-2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{K}\right)$$

moments about point A: $\quad \underset{\sim}{M} = \underset{\sim}{r}_{AD} \times \underset{\sim}{T} + \underset{\sim}{r}_{AF} \times \underset{\sim}{F} + \underset{\sim}{r}_{AB} \times \underset{\sim}{B} = \underset{\sim}{0}$

moment along axis AB: $\quad M_{AB} = \underset{\sim}{u}_{AB} \cdot \underset{\sim}{M} = \underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AD} \times \underset{\sim}{T}) + \underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AF} \times \underset{\sim}{F}) + \underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AB} \times \underset{\sim}{F})$

where, $\quad \underset{\sim}{u}_{AB} = \frac{\underset{\sim}{r}_B - \underset{\sim}{r}_A}{|\underset{\sim}{r}_B - \underset{\sim}{r}_A|} = \frac{6\underset{\sim}{j} - 4\underset{\sim}{i}}{\sqrt{52}} = -\frac{2}{\sqrt{13}}\,\underset{\sim}{i} + \frac{3}{\sqrt{13}}\,\underset{\sim}{j}$

$$\underset{\sim}{r}_{AD} = 2\underset{\sim}{j}$$

$$\underset{\sim}{r}_{AF} = -2\underset{\sim}{i} + 6\underset{\sim}{j}$$

note : $\quad \underset{\sim}{u}_{AB}$ is parallel to $\underset{\sim}{r}_{AB}$ so $\underset{\sim}{u}_{AB} \cdot (\underset{\sim}{r}_{AB} \times \underset{\sim}{F}) = \underset{\sim}{0}$

[$\underset{\sim}{F}_B$ passes through axis AB at B so creates no moment about the axis]

Problem 2

b.

$$M_{AB} = \begin{vmatrix} -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 0 \\ 0 & 2 & 0 \\ -\frac{2}{\sqrt{6}}T & -\frac{1}{\sqrt{6}}T & \frac{1}{\sqrt{6}}T \end{vmatrix} + \begin{vmatrix} -\frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 0 \\ -2 & 6 & 0 \\ 0 & 50 & -50\sqrt{3} \end{vmatrix} = 0$$

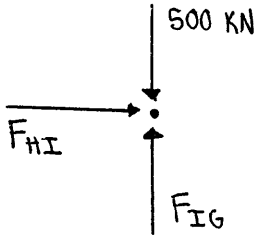
$$-\frac{2}{\sqrt{13}}\left[2\left(\frac{1}{\sqrt{6}}T\right)-0\right] - \frac{2}{\sqrt{13}}\left[6(-50\sqrt{3})-0\right] - \frac{3}{\sqrt{13}}\left[(-2)(-50\sqrt{3})-0\right] = 0$$

$$-\frac{4}{\sqrt{6}}T + 600\sqrt{3} - 300\sqrt{3} = 0$$

$$\boxed{T = 225\sqrt{2}\ \text{lb}}$$

Problem 3

a. joint I:

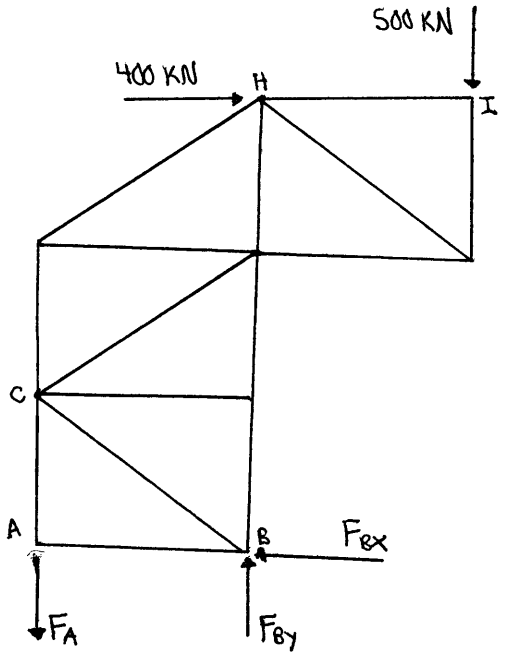


ΣF_y: $F_{IG} - 500 = 0$

$$\boxed{F_{IG} = 500\ \text{KN}\ (C)}$$

b. ΣF_x: $\boxed{F_{HI} = 0}$

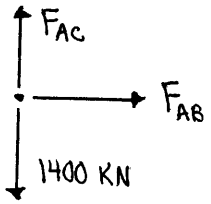
c.



$\circlearrowleft + \Sigma M_B$: $F_A(4) - 400(9) - 500(4) = 0$

$F_A = 1400\ \text{KN}$

joint A:



ΣF_y: $F_{AC} - 1400 = 0$

$$\boxed{F_{AC} = 1400\ \text{KN}\ (T)}$$