ME 133, Spring 2004, Test 1, 3/3/04

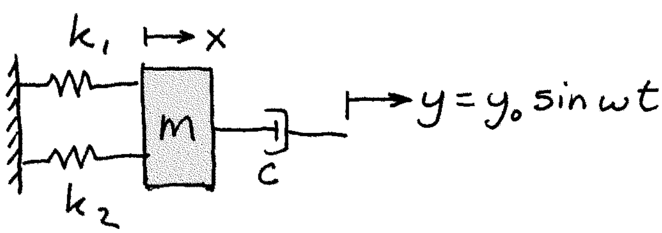
HONOR PLEDGE - I pledge my honor that during this examination I neither gave nor received assistance.

Name: ______________________________ Signed: ______________________________

1. Explicitly state what you're solving for and how. Box your answers. Except for problem 4, one word answers aren't acceptable. They'll get zero credit, whether correct or not. Demonstrate clearly that you know what you're doing - don't leave me guessing. These are general instructions. Now go on to the next item – the first real question.

2. Express $3\cos(2t) + 4\sin(2t)$ as $A\sin(2t + \phi)$.

3. What is the force across the damper for the illustrated system? $m = 10\,\text{kg}$, $k_1 = 400\,\text{N/m}$, $k_2 = 410\,\text{N/m}$, $c = 2\,\text{N·s/m}$, $\omega = 9\,\text{rad/s}$, $y_0 = 0.1\,\text{m}$. This problem can be solved by hand for a numerical answer.

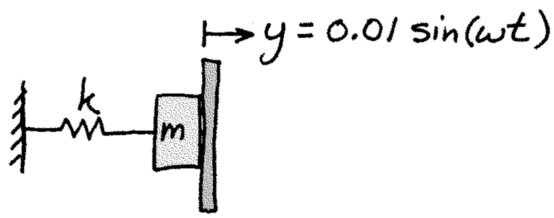


4. multiple choice: What is the natural frequency for the illustrated equation of motion?

$$2\ddot{x} - 72x = 0$$

   (a) 4 rad/s
   (b) 6 rad/s
   (c) 8 rad/s
   (d) none of the above

5. At what frequency will the mass lose contact with the floor? When $y = 0$ the spring is compressed by 2 cm. $m = 10\,\text{kg}$, $k = 1000\,\text{N/m}$.



SOLUTIONS – TEST 1
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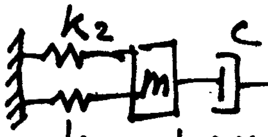
(2) $3\cos 2t + 4\sin 2t = A\sin[2t+\phi]$

$= A[\sin 2t\cos\phi + \cos 2t\sin\phi]$

$3 = A\sin\phi$, $4 = A\cos\phi$ $\Rightarrow$ $\tan\phi = 0.75 \Rightarrow \boxed{\phi = \tan^{-1}(0.75)}$

$A^2 = 3^2 + 4^2 \Rightarrow \boxed{A = 5}$

(3) 

$k \equiv k_1 + k_2$

$m\ddot{x} + c\dot{x} + kx = c\dot{y} = ci\omega y_o \cos\omega t$

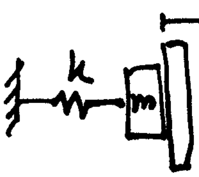
Let $x = x_o e^{i\omega t}$, forcing $= ci\omega y_o e^{i\omega t}$

$[k - m\omega^2 + i\omega c]x_o = ci\omega y_o$, $x_o = y_o \dfrac{ci\omega}{(k - m\omega^2 + i\omega c)}$

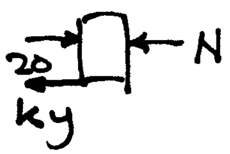
FORCE $= c(\dot{y} - \dot{x}) = ci\omega(y_o - x_o) = ci\omega\left(y_o - \dfrac{i\omega c y_o}{k - m\omega^2 + i\omega c}\right)$

$= ci\omega y_o \dfrac{(k - m\omega^2)}{(k - m\omega^2 + i\omega c)} = 2i(9)(0.1)\dfrac{(810 - 10(81))}{(810 - 10(81) + i(9)(2)} = \boxed{0}$

(4) $2\ddot{x} - 72x = 0 \Rightarrow \ddot{x} - 36x = 0$. THE STIFFNESS IS NEGATIVE. HENCE THERE ARE NO OSCILLATIONS, $x(t) = ae^{6t} + be^{-6t}$ $\boxed{(d)}$

(5) 

A 2cm compression MEANS THE STATIC FORCE IS $(1000)(0.02) = 20$ N



$m\ddot{y} = -ky + 20 - N$

$N = 20 - ky - m\ddot{y} = 20 - 1000(0.01\sin\omega t) + \omega^2(10)(0.01)\sin\omega t$

SEPARATION OCCURS WHEN $N = 0$. THUS

$20 = (1000 - 10\omega^2)(0.01)\sin\omega t$

FOR $\sin\omega t = 1.0$ WE HAVE $20 = (1000 - 10\omega^2)(0.01)$

$2000 = 1000 - 10\omega^2 \Rightarrow$ NO SOLUTION

FOR $\sin\omega t = -1.0$ WE HAVE $-20 = (1000 - 10\omega^2)(0.01)$

$3000 = 10\omega^2 \Rightarrow \boxed{\omega = \sqrt{300} \text{ rad/s}}$