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Spring 2008 *Professor Alaa Mansour*

Name: Date:

SID:

ME 107A Second Midterm Solution

1. The signal $y(t) = 10 \cos \omega t$ has a period of 5 seconds. Determine the following:
   a. The amplitude of the signal. (5 points)
   b. Its cyclic and circular frequencies. (5 points)
   c. The minimum sampling rate to avoid aliasing. (10 points)
   d. Its mean value over one period. (10 points)
   e. Its rms value over one period. (10 points)

Hint: $\int [\cos(ax)]^2 dx = \frac{1}{a}\left[-\frac{1}{2}\cos(ax)\sin(ax) + \frac{1}{2}ax\right]$

a. Amp. = 10.

b. Cyclic freq. $= f = \frac{1}{T} = \frac{1}{5} = \boxed{0.2\ \text{Hz.}}$

Circular freq.: $\omega = 2\pi f = 2\pi(0.2)$

$\boxed{\omega = 1.26\ \text{Rd/s}}$

c. $f_N = \frac{f_s}{2} \rightarrow f_s = 2 \cdot f_N = 2 \cdot 0.2\ \text{Hz}$

$\boxed{f_s = 0.4\ \text{Hz}}$

d. Mean: $\bar{y} = \frac{1}{T}\int_0^T y(t)\,dt$

$= \frac{1}{5}\int_0^5 10\cos(0.4\pi t)\,dt$ $\quad u = 1.26t$

$= \frac{\cancel{10}^{2}}{\cancel{5}} \cdot \frac{1}{1.26}\left[\sin(0.4\pi t)\right]_0^5$ $\quad \frac{du}{1.26} = dt$

$= \frac{2}{1.26}\left[\sin(2\pi) - \sin(0)\right] = 0 \rightarrow \boxed{\bar{y} = 0}$

e. $y_{rms} = \sqrt{\frac{1}{T}\int_0^T (y(t))^2\,dt}$

$= \sqrt{\frac{1}{5}\int_0^5 (10\cos(0.4\pi t))^2\,dt} = \sqrt{\frac{\cancel{100}^{20}}{\cancel{5}}\int_0^5 (\cos(\underbrace{0.4\pi}_{a} t))^2\,dt}$

$= \sqrt{20 \cdot \frac{1}{0.4\pi}\left[\frac{1}{2} \cdot 0.4\pi t - \frac{1}{2}\sin \cdot 0.4\pi t \cos 0.4\pi t\right]_0^5}$

$= \sqrt{\frac{20}{0.4\pi}\left[0.2\pi t - \frac{1}{2}\sin(0.4\pi t)\cos(0.4\pi t)\right]_0^5} = \sqrt{\frac{20}{0.4\pi}\left[\pi - \frac{1}{2}\sin 2\pi \cos 2\pi\right.}$ $\quad 5 - 0$

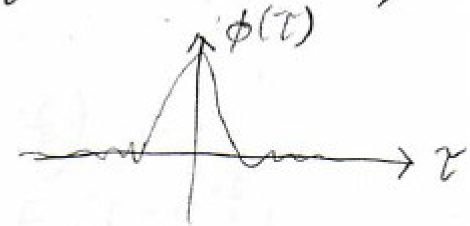
$\boxed{y_{rms} = 7.07}$

2. **A –** Define the auto correlation function of an ergodic random process and state two of its properties. (15 points)
   B – Which of the following are true?
   A single time history can be used to estimate the statistical properties of a process if the process is (a) deterministic, (b) ergodic, (c) stationary, (d) all of the above. (5 points)

A. Defn: $\phi(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} f(t) f(t+\tau)\, dt$

Properties:

1. $\phi(0) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} (f(t))^2\, dt$ = mean sq.
2. even function of τ : $\phi(+\tau) = \phi(-\tau)$
3. $\phi(0) \geq |\phi(\tau)|$



B. — (a) deterministic
(b) ergodic

3. A force transducer behaves as a second-order system. If the undamped natural frequency of the transducer is 1800 Hz and its damping is 30% of critical, determine the error in the measured force for a harmonic input of 950 Hz. (20 points)
   What would be the error for an input that has a frequency equal to the natural frequency? (20 points)

$$\frac{P_d}{P_s} - 1 = \frac{1}{\sqrt{\left[1-(f/f_n)^2\right]^2 + (2\zeta f/f_n)^2}} - 1$$

$$f = 950\,Hz \qquad f_n = 1800\,Hz$$

$$\zeta = .30$$

$$\frac{P_d}{P_s} - 1 = .27 \approx 27\% \text{ Error}$$