

Engineering 11  
Fall 2003  
James Hunt

Name KEY

Quiz No. 1  
(October 10, 2003)

1 ( 25 pts) \_\_\_\_\_

2 (25 pts) \_\_\_\_\_

3 (25 pts) \_\_\_\_\_

4 (25 pts) \_\_\_\_\_

The exam is closed books and notes.

Do not ask for clarification of the problem statements, part of the exam is understanding the questions.

If you think there is an error in the problem, state any necessary assumptions and proceed.

Partial credit is given for partially correct work.

1. Answer the following questions in the space provided.

- ⑤ (a) List one air quality contaminant, indicate its source, and state why it is a contaminant. (you may not use lead)

SO<sub>x</sub>  
NO<sub>x</sub>  
PM<sub>10</sub> / PM<sub>2.5</sub>  
O<sub>3</sub>  
CO

- ⑤ (b) List one water quality contaminant, indicate its source, and state why it is a contaminant. (you may not use perchlorate)

many

- ⑤ (c) List one contaminant of the land surface, indicates its source, and state why it is a contaminant.

many

1 (cont.)

⑤ (d) Why was lead removed from gasoline?

So that catalytic converters would work.

⑤ (e) List three ways of increasing the fuel efficiency of gasoline-powered automobiles.

Reduce weight

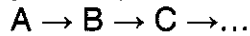
Reduce coef. of friction for tires

Reduce coef. of drag

Reduce projected area

Reduce velocity

2. Radioactive materials undergo first order decay (the decay rate is proportional to the atoms present). Consider a decay chain series of three radioisotopes: A, B, and C:



- ⑤ (a) If there exists  $A_0$  atoms initially, derive the expression for the number of atoms of A as a function of time. Assume the first order decay rate constant is  $k_A$ .



$$\text{IN} = \text{OUT} + \text{ACCUMUL.}$$

$$0 = k_A A \Delta t + \Delta A$$

-3 deriv

$$\frac{dA}{dt} = -k_A A$$

Integrate

$$A = A_0 e^{-k_A t}$$

- ⑤ (b) What is the expression for the half life of A?

$$\text{at } t = t_{1/2} \quad A = \frac{1}{2} A_0$$

$$\frac{1}{2} A_0 = A_0 e^{-k_A t_{1/2}}$$

$$\ln 2 = k_A t_{1/2}$$

$$t_{1/2} = \frac{\ln 2}{k_A}$$

- ⑤ (c) Derive a differential equation whose solution would predict the number of atoms of B. Let  $k_B$  represent the first order decay rate constant for B.



$$\text{IN} = \text{out} + \text{accumul.}$$

$$k_A A \Delta t = k_B B \Delta t + \Delta B$$

$$\frac{dB}{dt} = k_A A - k_B B$$

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⑤

2 (cont.)

- ⑤ (d) A hazardous waste facility received a container having an unknown amount of isotope A within it. After the container sat around for 10 days, the container was measured to have a radioactive decay rate of  $10^3$  atoms of A per second. The container was counted again after being at the facility for 20 days and found to have a decay rate of  $10^2$  atoms of A per second. What is the first order decay rate constant for isotope A?

$$t = 10d \quad 10^3 = A_0 k_A e^{-k_A(10d)}$$

$$t = 20d \quad 10^2 = A_0 k_A e^{-k_A(20d)}$$

divide 2 eqn's:  $10 = e^{-k_A(10-20)}$

Solve  $k_A = \frac{\ln 10}{10d} = 0.23 d^{-1}$   
 $= 2.7 \times 10^{-6} s^{-1}$

$$\text{decay rate at time } t = \left. \frac{dA}{dt} \right|_t = \frac{d}{dt} (A_0 e^{-k_A t}) = -A_0 k_A e^{-k_A t}$$

- ⑤ (e) For the conditions in (d), how many atoms of A were initially delivered to the hazardous waste facility?

use  $k_A$  from (d), plus into one of the equations.

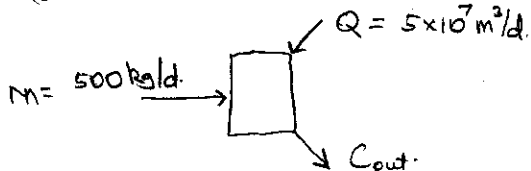
$$10^3 = A_0 k_A e^{-k_A(10d)}$$

$$A_0 = \frac{10^3 (\#/s) (3600 \cdot 24 \text{ h/d})}{k_A e^{-k_A(10d)}}$$

$$A_0 = 3.7 \times 10^9 \text{ atoms.}$$

3. Perchlorate has been introduced into the Colorado River system from the manufacturing system in Henderson, Nevada. The manufacturing facility has been discharging perchlorate at a rate of 500 kg/day for 30 years. As you recall, perchlorate dissolved in water is inert and can be modeled as a conservative, nonreactive compound. The flow rate in the Colorado River leaving Nevada is  $5 \times 10^7 \text{ m}^3/\text{day}$ .

- ① (a) What is the concentration of perchlorate in the Colorado River leaving Nevada?



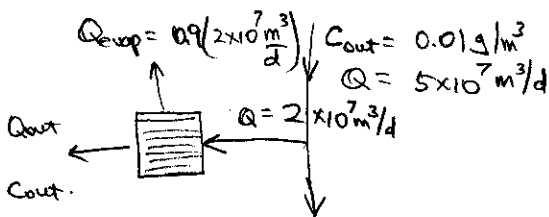
$$\text{mass in} = \text{mass out} \quad (\text{Steady State})$$

$$500 \text{ kg/d} = C_{\text{out}} (5 \times 10^7 \text{ m}^3/\text{d})$$

$$C_{\text{out}} = \frac{(500 \text{ kg/d}) (1000 \text{ g/kg})}{5 \times 10^7 \text{ m}^3/\text{d}}$$

$$C_{\text{out}} = 0.010 \text{ g/m}^3$$

- ② (b) After the Colorado River flows into California,  $2 \times 10^7 \text{ m}^3/\text{day}$  of water is diverted for irrigation where 90% of that water undergoes evaporation and transpiration. What is the perchlorate concentration in the irrigation return flow?



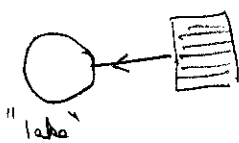
$$\text{Water Balance for farm. } Q_{\text{irr. ret.}} = 0.20 \times 10^7 \text{ m}^3/\text{d}$$

$$\text{Perchlorate balance } (0.01 \text{ g/m}^3)(2 \times 10^7 \text{ m}^3/\text{d}) = C_{\text{irr. ret.}} (0.20 \times 10^7 \text{ m}^3/\text{d})$$

$$\text{Solving } C_{\text{irr. ret.}} = (0.01 \text{ g/m}^3) \frac{2 \times 10^7 \text{ m}^3/\text{d}}{0.2 \times 10^7 \text{ m}^3/\text{d}}$$

$$= 0.10 \text{ g/m}^3$$

- ③ (c) The irrigation return flow is dumped into a terminal lake with no outlet. The lake has a steady state water volume of  $10^9 \text{ m}^3$ . What is the perchlorate concentration in the lake?



mass perchlorate added to lake

$$= (0.10 \text{ g/m}^3) (0.2 \times 10^7 \text{ m}^3/\text{d}) \left( \frac{365 \text{ d}}{\text{yr}} \right) (30 \text{ yr})$$

$$= 2.2 \times 10^9 \text{ g}$$

$$\text{Perchlorate Conc.} = \frac{\text{mass}}{\text{Volume}} = \frac{2.2 \times 10^9}{10^9 \text{ m}^3}$$

$$= 2.2 \text{ g/m}^3$$

4. Under some circumstances, population growth is modeled as constant and independent of the current population:

$$\frac{dP}{dt} = R$$

where  $P$  is the population and  $R$  is the growth rate.

- ⑧ (a) If the initial population is  $P_0$ , what is the expression for the population as a function of time?

$$\frac{dP}{dt} = R$$

integrate.

$$\int_{P_0}^P dP = R \int_0^t dt \Rightarrow P - P_0 = Rt \Rightarrow \boxed{P = P_0 + Rt}$$

⑧ Proportional.

(b) One extension for this growth model is to include population decay (death) that is proportional to the population. Let  $k_D$  represent the first order decay rate constant. What is the differential equation that describes population under these conditions?



in = out + accuml.

$$R \Delta t = k_D P \Delta t + \Delta P$$

$$\frac{dP}{dt} = R - k_D P$$

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- ⑧ (c) What is the expression for the maximum population under the conditions in (b)?

$P_{max}$  occurs when growth balanced by decay

$$\frac{dP}{dt} = 0 \Rightarrow R = k_D P_{max} \Rightarrow P_{max} = \frac{R}{k_D}$$

- ① (d) Speculate on what might cause constant growth rates.

immigration limitation (us had limit)

transportation limitation (gold rush.)