CS184, Fall 1997
Midterm #1
Professor Brian A. Barksy

Fall 1997	Foundations of Computer Graphics	Professor Brian A. Barsky and Adrian Sfarti
October 9, 1997	CS184	T.A.'s: Lilian Chu, Dan Garcia and Aaron Isaksen

MIDTERM EXAM

NAME (LAST, FIRST):
CS184-:

Relax. You have 80 minutes. Remember to pace yourself. Feel free to use the back of each page for additional answer space. Note that not all questions have equal value. Do not panic.

GOOD SKILL!

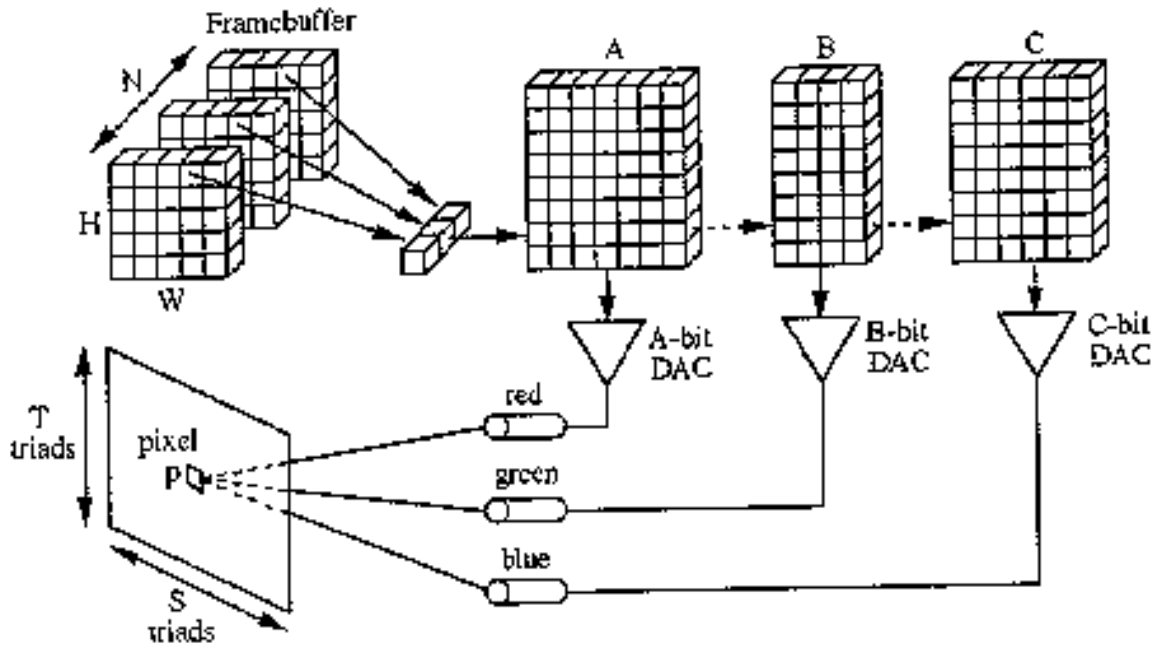(write any comments about the exam here)

How well do you believe you did?
(Circle the appropriate picture which represents your perceived midterm rank)

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Question	Possible	Your Score

1	15
2	25
3	25
4	35
Total	100

Question 1: Hardware and Scan Conversion (15 points)

The diagram shown above shows a colormap system with a N bit deep W wide and H high framebuffer, digital-to-analog converters (DAC) for red, green
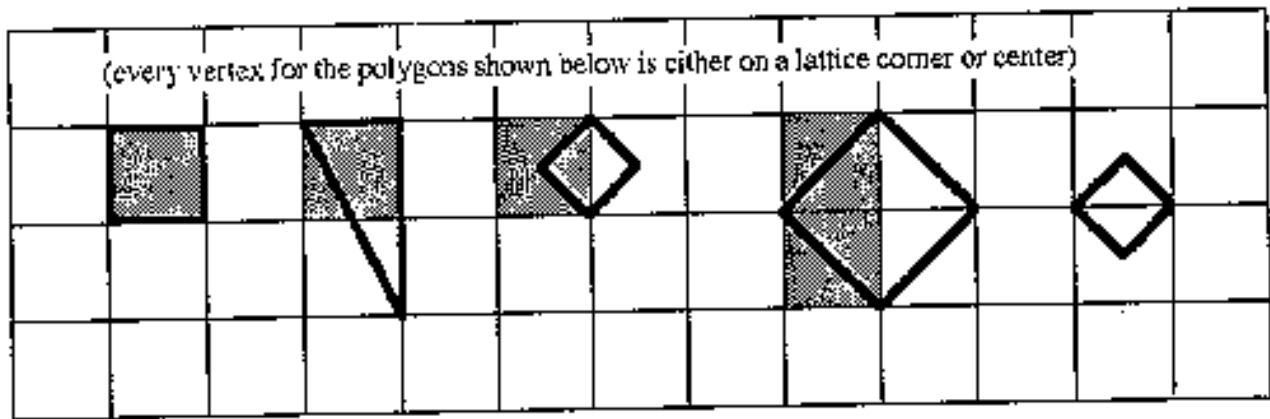
1.1) ...the # of pixels on the screen. [1 pt]

and blue which are A, B and C bits wide respectively, and a screen with S triads wide and T triads high. Using the variables above (and assuming nothing about N, W, H, A, B, C, S or T - the diagram above is not to scale), **derive an expression for:**

1.2) ...the # of colors that could ever land on pixel p over time. [2 pts]

1.3) ...the # of possible colors on the screen at once. [2 pts]

(every vertex for the polygons shown below is either on a lattice corner or center)

1.4) Up to now, you have always been given a sample point (and two tie-breaking directions) and asked to render polygon primitives. Here we give you the polygon primitives and rendered pixels and ask you to figure out the sample point and two tie-breaking directions. **The sample point lies on the edge of the polygon.** Hint: The tie-breaking driections **do not necessarily have to be horizontal or vertical.** [10 pts]

Answer 1: Hardware and Scan Conversion

1.1)

1.2)

1.3)

1.4) (Indicate sample point and two tie breaking directions on the picture below.)

Question 2: Transformations (25 points)

The figure to the right shows a 2-D triangle **PQR** transformed by a transformation matrix **T** to a new position **P''Q''R''**. Pay careful attention to the fact that **T** must transform **P** to **P''**, **Q** to **Q''** and **R** to **R''**.

2.1) Using row-vector notation, show **T** as a composition of matrices **Scale(s_x, s_y), Translate(t_x, t_y), Rotate(theta), Shear$_x$ (a)** or **Shear$_y$(b)** with appropriate arguments. [10 pts]
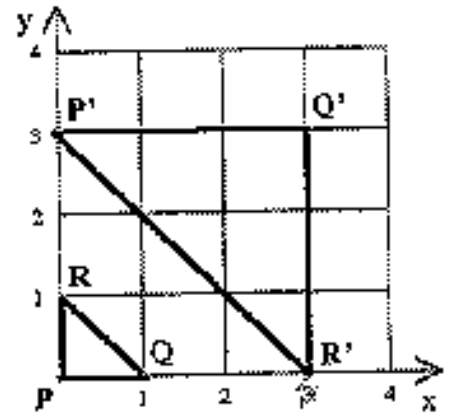
E.g., **T = Rotate(30°) * Shear$_x$(5)*Translate(10,3)**

2.2) Write down the 3x3 transformation matrix **T**. Show all work. [10 pts]

2.3) Show the inverse matrix **T^{-1}** (which transforms the triangle **P''Q''R''** to **PQR**) using a composition of matrices **Scale(s_x,s_y),**

Translate (t_x, t_y), Rotate(theta), Shear$_x$(a) or **Shear$_y$ (b)** with

appropriate arguments. You may find your answer to question 2.1 useful here. [5 pts]
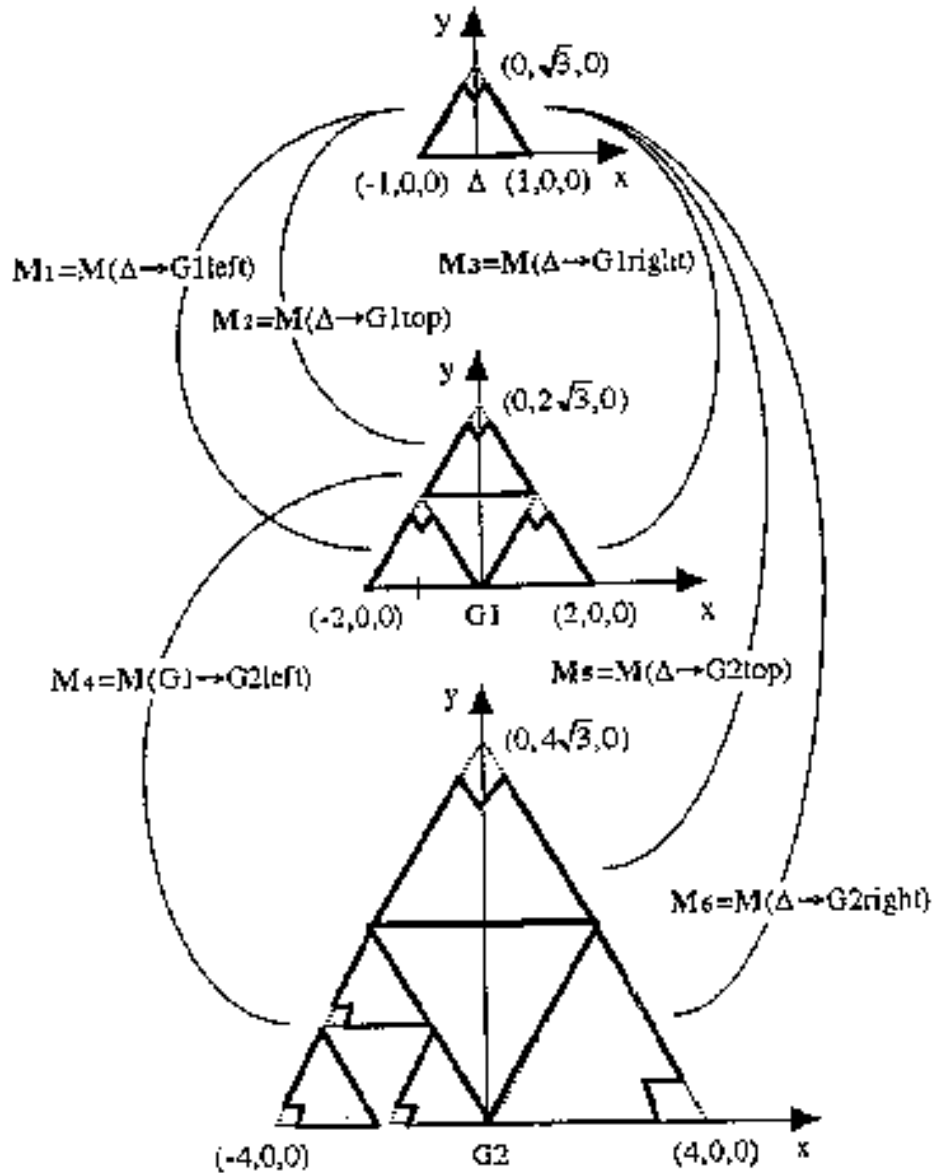
Answer 2: Transformations

2.1) T =

2.2)

2.3) T^{-1} =

Question 3: Hierarchies and Change of Bases (25 points)

This diagram show how the upper triangle can be instanced five times to form the middle group G1 (G1left, G1top and G1right) and part of the bottom group G2 (G2top and G2right). G1 can be instanced once to help form the bottom group G2 (component G2left). Coordinates refer to the verices of the subtending triangle.

3.1) Find the 6 transformations labeled M_1 thorugh M_6 in the diagram above. You do NOT need to create the matices for the transformations, you can just write them as products of row vector transformations **Scale (s_x, s_y), Translate(t_x, t_y), Rotate(theta), Shear$_x$(a)** or **Shear$_y$(b)** with appropriate arguments. [6 pts]

3.2) Draw the hierarchy tree, labeling the branches with the appropriate concatenated row-vector transformations M_1 through M_6. Show the state of the stack entering each node. [15 pts]

Captain Matrix takes your ordinary **X, Y,** and **Z** axes, rotates them about the origin in some unknown way until he obtains three new axes, **a, b,** and **c,** such that a used to be **X, b** used to be **Y,** and **c** used to be **Z.** He then politely gives you the change of basis matrix such that the expression to the right holds:

$$\begin{bmatrix} a_x & a_y & a_z & 0 \\ b_x & b_y & b_z & 0 \\ c_x & c_y & c_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} o_x \\ o_y \\ o_z \\ 1 \end{bmatrix} - \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}$$

3.3) What are the **numerical** values of e, f, g, and h? [4 pts]

Answer 3: Hierarchies and Changes of Bases

3.1)
M_1=M(delta->G1left)=

M_2=Mdelta->G1top)=

M_3=M(delta->G1right)=

M_4=M(G1->G2left)=

M_5=M(delta->G2top)=

M_6=M(delta->G2right)=

3.2)
3.3)
e=
f=
g=
h=

Question 4: Projection (35 points)

Your hovercraft floats above your lake as in assignment 4. Here you decide to place the center of projection (designated by the eye in the diagram below) on the hovercraft (not shown) itself. **There is NO clipping.** Your projection and geometry parameters:

- The origin is designated by the label O near the small axes in each diagram.
- Center of projection = (2,0,1)
- The lake is defined by A (0,-1,0) B(3,-1,0) C(3,-1,3) and D(0,-1,3)
- Projection plane is the z=5 plane
- Point C after projection and homogenous division maps to **world coordinate** (4,-2,5) as shown below on the projection plane z=5 in world coordinates (not viewing coordinates).
- Each grid line in all of the figures below is one unit (including the answer plot on the bottom)

TOP VIEW (view from +y)   SIDE VIEW (view from +x)   REAR VIEW (view from -z)
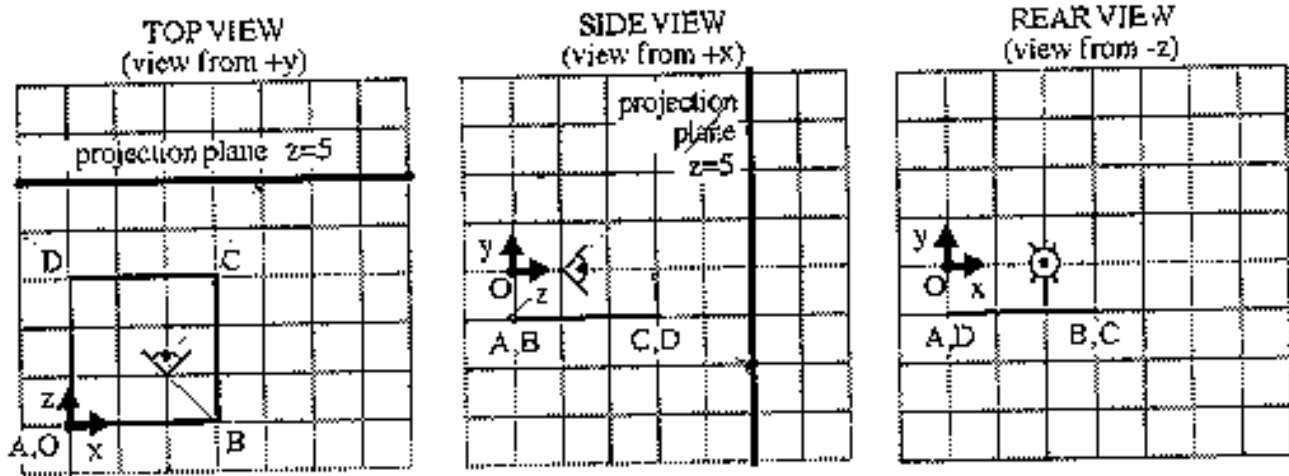
4.1) Project A, B and D onto the projection plane diagram z=5 shown on the bottom left on the previous page. We've already projected C for you. Then copy your answer to the right diagram (you'll use one diagram for the answer to 4.6 and one for the answer to 4.7 below). [3 pts]

4.2) Write down the row-vector notation tranformation matrix (you may express it as a single 4x4 matrix or as a composition of 4x4 matrices) that, after homogeneous division, has the effect of projecting ABCD onto the projection plane z=5. HINT: You may wish to verify it with point C whose coordinates are given before and after projection.

4.3) Assuming ABCD was the bottom face of a cube, *what classification of projection is this?* List the entire hierarchy (E.G., planar-parallel-orthographic-axonometric-trimetric) [3 pts]

4.4) *How many* vanishing points are there with this projection? [2 pts]

4.5) *Where* are the vanishing points? [4 pts]

4.6) Let's say we project points ABCD onto the projection plane (as you have done in 4.1) and then call a simple 2D polygon **wireframe** rendering function, *draw polygon ABCD as it would be rendered* on the previous page's lower-left projection plane. [4 pts]
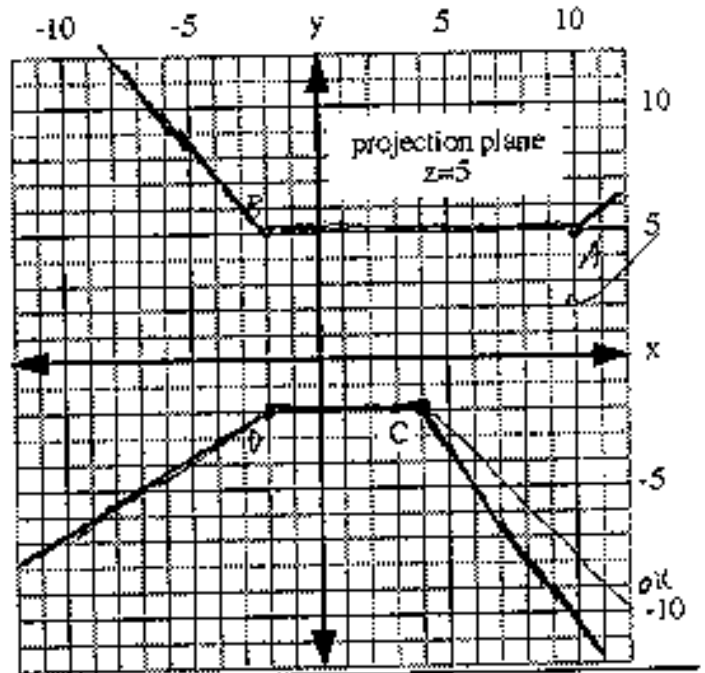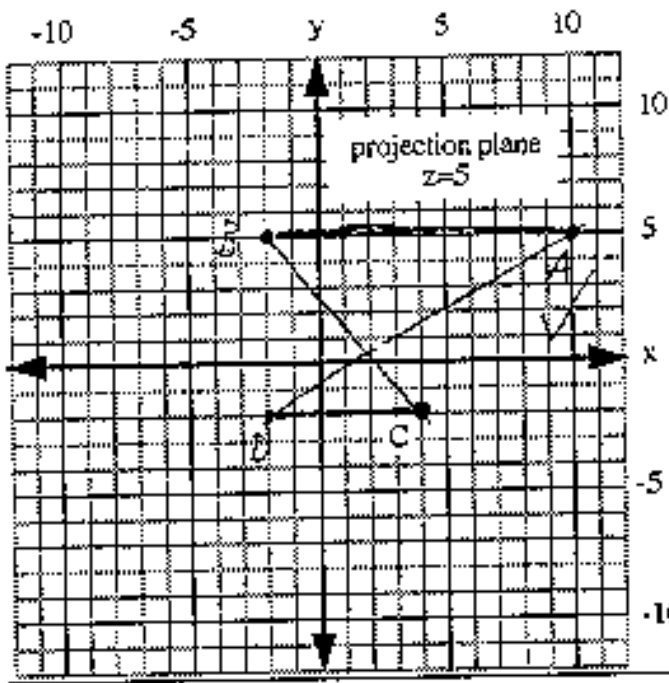
4.7) You may notice something strange about your answer for 4.6. Let's see if we can solve that. Imagine a fly flying along the edges of your lake in 3D **before projection** (straight from A to B to C to D and back to A). Trace out the path of the fly you would see in world coordinates on the projection plane z=5. Draw your answer on the previous page's lower-right plane. [4 pts]

Answer 4: Projection

Plot your answers to questions 4.1 and 4.6 below.

Note: ignore writing on the graphs: resulted from scanned images

Plot your answers to questions 4.1 and 4.7 below.

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4.2)

4.3)

4.4)

4.5)