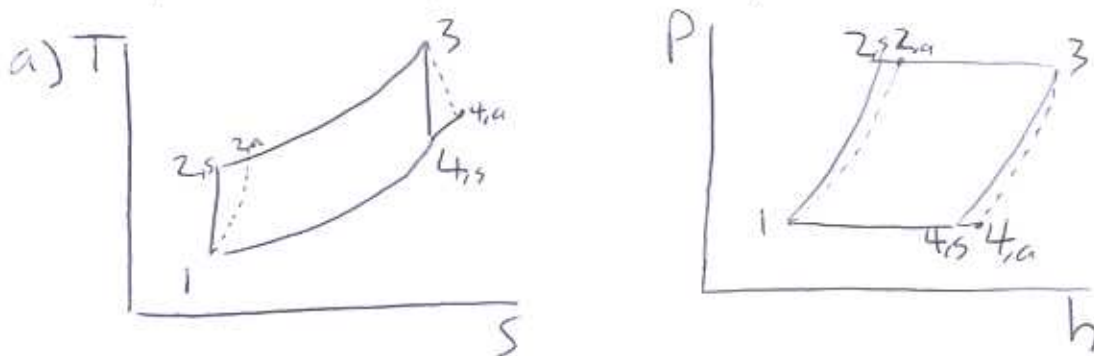


MIDTERM #2

1. (35 pts.) An Brayton cycle is run with incoming air at atmospheric pressure and a temperature of 25 °C, and a pressure ratio of 18.39. The heat addition downstream of the compressor is 445.62 kJ/kg air. The turbine outlet pressure is 1 atmosphere. If the compressor and turbine are both 75% efficient:

- draw the cycle on T-s and P-v diagrams
- calculate the maximum temperature (do **not** assume a constant specific heat)
- calculate the net work produced
- what is the thermodynamic efficient of this cycle?



b) $\frac{P_2^0}{P_1^0} = \frac{P_2}{P_1}$ $P_1^0 = 1.3715$ from table 5s

$P_2^0 = P_1^0 \frac{P_2}{P_1} = 25.22 \rightarrow h_{2,s} = 685.5 \frac{\text{kJ}}{\text{kg}}$ from table 5s
 $h_1 = 299.03 \frac{\text{kJ}}{\text{kg}}$ " " "

$\eta_c = \frac{h_{2,s} - h_1}{h_{2,a} - h_1} = 0.75$ $h_{2,a} = \frac{h_{2,s} - h_1}{0.75} + h_1 = 814.3 \frac{\text{kJ}}{\text{kg}}$

$Q_{in} = h_3 - h_2 = 445.62 \frac{\text{kJ}}{\text{kg}} \rightarrow h_3 = 1259.9 \frac{\text{kJ}}{\text{kg}}$

$T_{max} = T_3 = 910.66^\circ\text{C}$ From table 5s

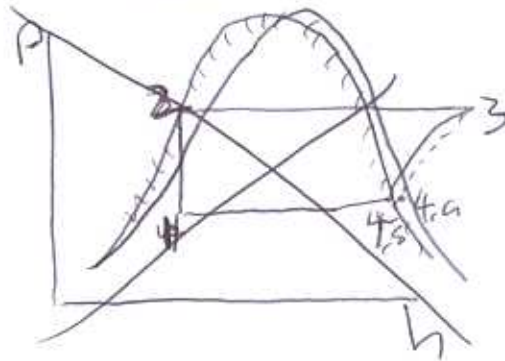
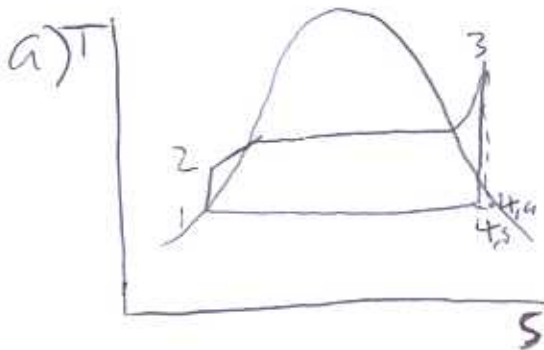
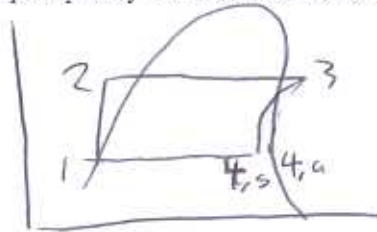
c) $W_{net} = h_3 - h_4 - (h_2 - h_1)$ $\frac{P_3}{P_4} = \frac{P_3^0}{P_4^0}$ $P_3^0 = 228.624$
 $P_4^0 = P_3^0 \frac{P_4}{P_3} = 12.43 \rightarrow h_{4,s} = 559.64$

$\eta_t = \frac{h_3 - h_{4,a}}{h_3 - h_{4,s}} = 0.75$ $h_{4,a} = 0.75(h_3 - h_{4,s}) + h_3 = 734.71$

$W_{net} = 9.944 \frac{\text{kJ}}{\text{kg}}$ d) $\eta = \frac{W_{net}}{Q_{in}} = \frac{9.944}{445.62} = 0.0223 = 2.2\%$

2. (35 pts.) A solar-driven water vapor Rankine cycle is considered for residential electrical energy generation. The heat into the solar collector on a clear sunny day is 10 kW. The flow rate of water is set such that the maximum temperature is 300 °C; the maximum pressure is 200 kPa. The turbine is 75% efficient and the ambient temperature is 20 °C. Downstream of the turbine is a condenser that operates at ambient temperature. The water leaves the condenser with zero quality. The pump may be assumed have an efficiency of 100%

- Draw the process on T-s and P-h diagrams
- Calculate the water flow rate (kg/s).
- What is the net power from the cycle?
- What is the thermal efficiency of this cycle?



$$b) \dot{Q}_{in} = \dot{m}(h_3 - h_2) = 10 \text{ kW}$$

$$h_1 = 83.835 \frac{\text{kJ}}{\text{kg}} \quad v_1 = 0.001002 \frac{\text{m}^3}{\text{kg}} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{From Table}$$

$$h_2 = h_1 + v \Delta P \approx 84 \frac{\text{kJ}}{\text{kg}} \quad h_3 = 3071.4 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{m} = \frac{10 \text{ kW}}{3071.4 \frac{\text{kJ}}{\text{kg}} - 84 \frac{\text{kJ}}{\text{kg}}} = 3.3 \times 10^{-3} \frac{\text{kg}}{\text{s}}$$

$$c) \dot{W}_{net} = \dot{Q}_H - \dot{Q}_C = \dot{W}_t - \dot{W}_p = \dot{m} [(h_3 - h_2) - (h_4 - h_1)]$$

$$s_{4,s} = s_3 = 7.893 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \rightarrow h_{4,s} = 2309.09 \frac{\text{kJ}}{\text{kg}} \quad \text{From Table}$$

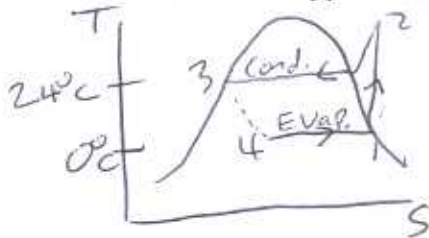
$$\eta_t = \frac{h_3 - h_{4,s}}{h_3 - h_{4,c}} = 0.75 \quad h_{4,c} = h_3 - 0.75(h_3 - h_{4,s}) = 2500 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{W}_{net} = 10 \text{ kW} - \dot{m}(h_4 - h_1) = 2.03 \text{ kW}$$

$$d) \eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{2.03 \text{ kW}}{10 \text{ kW}} = 20.3\%$$

3. (30 pts) A heat pump is used to provide space heat. The compressor is run by a 2000 kW motor, the evaporator temperature is 0 °C, and the condenser temperature is 26.72 °C. For a compressor efficiency of .8, and R134a qualities of 0 and 1 leaving the condenser and evaporator, respectively:

- What is heat transfer rate from the condenser (kW)?
- What is the coefficient of performance?
- Briefly discuss the energy efficiency of the use of a heat pump versus a gas-fired furnace for this application.



NOTE: This problem is overspecified, so there are two valid solutions!

Solution 1

$$a) \dot{Q}_H = \dot{m}(h_2 - h_3)$$

$$h_1 = 247.23 \frac{\text{kJ}}{\text{kg}}$$

$$\left. \begin{aligned} h_3 &= 86.78 \frac{\text{kJ}}{\text{kg}} \\ s_{2,s} = s_1 &= 0.9190 \frac{\text{kJ}}{\text{kg}} \\ h_{2,s} &= 265.37 \frac{\text{kJ}}{\text{kg}} \end{aligned} \right\} \text{From table}$$

$$\eta_c = \frac{h_{2,s} - h_1}{h_{2,a} - h_1} = 0.8$$

$$h_{2,a} = \frac{h_{2,s} - h_1}{0.8} + h_1 = 269.9 \frac{\text{kJ}}{\text{kg}}$$

if $\dot{m} = 22 \text{ kg/s}$

$$\dot{Q}_H = 22 \text{ kg/s} (269.9 - 86.78) \frac{\text{kJ}}{\text{kg}} = \boxed{4,029 \text{ kW}}$$

$$b) \text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = \frac{\dot{m}(h_2 - h_3)}{\dot{m}(h_2 - h_1)} = \boxed{2.01}$$

Solution 2

~~! a) Values from table still correct but calculations need \dot{m} from given compressor work!~~

~~$\dot{W}_c = \dot{m}(h_2 - h_1)$ $\dot{m} = \frac{2000 \text{ kW}}{26}$~~

Same values from table

$$\left\{ \begin{aligned} h_1 &= 247.23 \frac{\text{kJ}}{\text{kg}} \\ h_3 &= 86.78 \frac{\text{kJ}}{\text{kg}} \end{aligned} \right. \quad 2000 = \dot{W}_c = \dot{m}(h_2 - h_1) \quad h_2 = \frac{\dot{W}_c}{\dot{m}} + h_1 = 338.14 \frac{\text{kJ}}{\text{kg}}$$

$$\dot{Q}_H = \dot{m}(h_2 - h_3) = 22 \frac{\text{kg}}{\text{s}} (338.14 - 86.78) \frac{\text{kJ}}{\text{kg}} = \boxed{5,529.9 \text{ kW}}$$

$$b) \text{COP}_{\text{HP}} = \frac{\dot{Q}_H}{\dot{W}_{\text{net}}} = 2.776$$

(c) Since η_{th} is usually 30-40%, for a power cycle, a heat pump must have a COP of greater than $\frac{1}{3} - \frac{1}{4}$ to have a greater efficiency than burning the gas directly, where all of the thermal energy can be used.