QUESTION # 1 ME 105 Name ANSWER KEY

AVG $\frac{17.1}{30} \Rightarrow 57.1\%$ STD: 5.88

ME 105 Thermodynamics

First Midterm Exam – 50 MINUTES

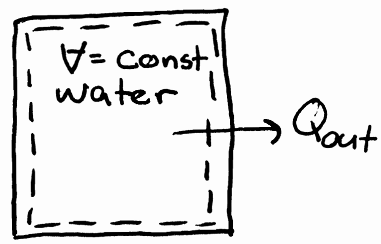
Three questions, equal weight.

YOU ARE RESPONSIBLE TO WRITE YOUR NAME ON EVERY PAGE. WE WILL SEPARATE THE EXAM PAGES AND PAGES WITHOUT NAME WILL NOT BE GRADED. WRITE ALSO THE NUMBER OF THE QUESTION ON EACH PAGE

Question #1 (easy question)

Two kilograms of steam at a pressure of 100 kPa is contained in a rigid sealed tank whose volume is 3.97 m^3. The steam begins to cool as heat is transferred to the atmosphere. When the internal pressure reaches 10 kPa, the tank wall will collapse.

[2] a) Sketch a P-V diagram of the process
[2] b) What is the initial temperature in the tank?
[2] c) What will be the temperature in the tank when the walls collapse?
[2] d) How many kg of water are in the liquid state at the instant of collapse.
[2] e) What is the total change in the internal energy for the process.



a) $m = 2\,kg$ $P_1 = 100\,kPa$

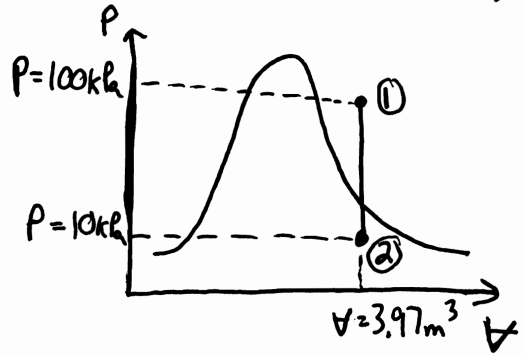
$\forall = 3.97\,m^3 = \text{constant}$ $v_1 = \frac{\forall}{m} = \frac{3.97\,m^3}{kg} = 1.958\,m^3/kg$

From steam tables, (A-6), state ① is superheated water

$P_2 = 10\,kPa$

$v_2 = v_1 = 1.958\,m^3/kg$

From saturated water table, (A-5), state ② is in the saturated region



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b) From table (A-6) with $P_1 = 100\,kPa$, $v_1 = 1.9538 \frac{m^3}{kg}$

$$T_1 = 150^\circ C + \frac{(1.985 - 1.9367)\,\cancel{m^3/kg}}{(2.1724 - 1.9367)\,\cancel{m^3/kg}}(200-150)^\circ C$$

This is linear interpolation

$\boxed{T_1 = 160.2^\circ C}$

c) $P_2 = 10\,kPa$, from part a), in saturated region

Therefore, $T_2 = T_{sat}$ at $P_{SAT} = 10\,kPa$, $\boxed{T_2 = 45.81^\circ C}$ (A-5)

d) $x = \frac{m_{vapor}}{m_{total}} = \frac{m_{total} - m_{liquid}}{m_{total}} = 1 - \frac{m_{liquid}}{m_{total}} \Rightarrow m_{liquid} = m_{total}(1-x)$

$x = \frac{v_{avg} - v_f}{v_{fg}} = \frac{v_2 - v_f}{v_{fg}}$ in this case

$m_{liquid} = m_{total}\left(1 - \frac{v_2 - v_f}{v_{fg}}\right)$

where $m_{total} = 2\,kg$ (given)

(A-5) $P_{SAT} = 10\,kPa$ $\left\{ \begin{array}{l} v_f = 0.001010\,m^3/kg \\ v_{fg} = 14.669\,m^3/kg \end{array} \right.$

$v_2 = v_1 = 1.985\,m^3/kg$

$= 2\,kg\left(1 - \frac{(1.985 - 0.001010)\,\cancel{m^3/kg}}{14.669\,\cancel{m^3/kg}}\right)$

$\boxed{m_{liquid} = 1.730\,kg}$

e) $\Delta U = m(u_2 - u_1)$

$u_2 = u_f + x u_{fg}$ at $P_{sat} = 10\,kPa$ (A-5)

$u_2 = 191.79 \frac{kJ}{kg} + \underbrace{\left(\frac{(1.985 - 0.001010)\,\cancel{m^3/kg}}{14.669\,\cancel{m^3/kg}}\right)}_{\text{"x"}}(2245.4)\frac{kJ}{kg}$

$u_2 = 495.482 \frac{kJ}{kg}$

u_1 can be found from linear interpolation of data from table A-6. At $P_1 = 100\,kPa$, $v_1 = 1.985\,m^3/kg$

$u_1 = 2582.9 \frac{kJ}{kg} + \left(\frac{1.985 - 1.9367}{2.1724 - 1.9367}\right)\frac{\cancel{m^3/kg}}{\cancel{m^3/kg}}(2658.2 - 2582.9)\frac{kJ}{kg}$

$u_1 = 2598.33 \frac{kJ}{kg}$

$\Delta U = m(u_2 - u_1) = 2\,\cancel{kg}(495.482 - 2598.33)\frac{kJ}{\cancel{kg}} = \boxed{-4205.7\,kJ = \Delta U}$

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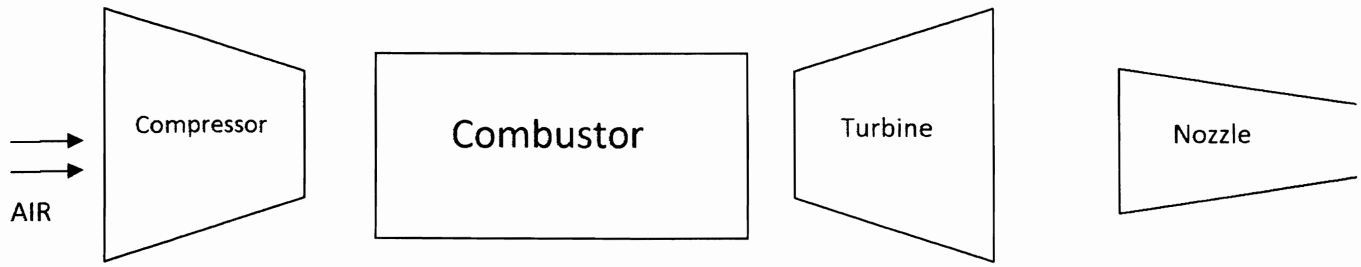
Question # 2 (difficult question)



A Simplified Jet Engine is to be analyzed.

Air enters an adiabatic compressor on the intake side of the jet engine at atmospheric pressure and a temperature of 25°C. Air exits the compressor at a temperature of 127°C and goes into a combustor. The combustor can be modeled as a heat exchanger which inputs heat into a system at a rate of 1500 kJ/s. The stream coming out of the combustor is sent into an adiabatic turbine where work is extracted. After the air exits the turbine it is at a temperature of 625°C. The air is then passed through a nozzle with an exit cross-sectional area of 30 cm^2 and exhausted to the environment at atmospheric pressure and 25°C.

You may assume the air behaves as an ideal gas and maintains an average constant specific heat of 1.07 kJ/kg K throughout the entire engine. You may also assume that all of the work produced by the turbine goes into operating the compressor. The gas constant of air is 0.287 kJ/kgK, atmospheric pressure is 101.3 kPa.

Find:

A) The temperature of the stream after it exits the combustor
B) The mass flow rate of air through the system
C) The power produced by the turbine
D) The exit exhaust velocity

Assumptions: 1. Air behaves as ideal gas

2. $C_{P_{avg}} = C_p = 1.07 \frac{KJ}{KgK}$ for air throughout engine

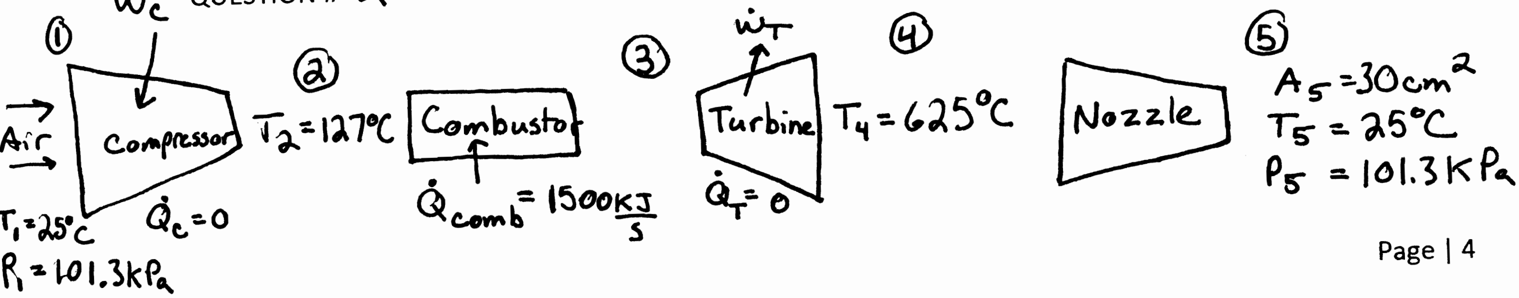
3. $\dot{W}_T = \dot{W}_C$

4. $R = 0.287 \frac{KJ}{KgK}$ for air

5. Steady flow

6. Combuster: KE, PE, $\dot{W} = 0$

9. $\dot{Q}_C = \dot{Q}_T = 0$



For each component, there is one inlet and 1 exit. Thus, for each Component, the mass conservation is:

$$\dot{m}_{in} - \dot{m}_{out} = \cancel{\frac{dm_{cv}}{dt}}^{0} \Rightarrow \dot{m}_{in} = \dot{m}_{out} = \dot{m}$$ ← constant throughout.

a) Energy balance for combustor:

$$\dot{E}_{in} - \dot{E}_{out} = \cancel{\frac{d\dot{E}_{cv}}{dt}}^{0} \Rightarrow \dot{E}_{in} = \dot{E}_{out}$$

[1.5] $\dot{m}h_2 + \dot{Q}_{comb} = \dot{m}h_3 \Rightarrow \dot{m}(h_3 - h_2) = \dot{Q}_{comb} \Rightarrow \dot{m}c_p(T_3 - T_2) = \dot{Q}_{comb}$

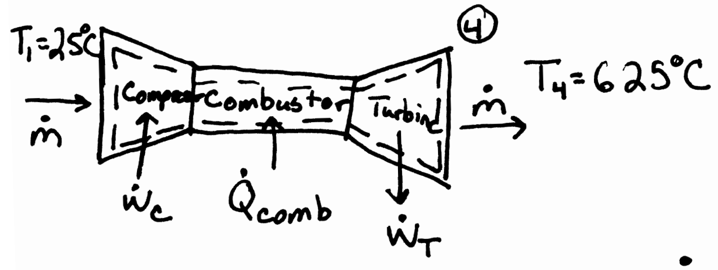
$$T_3 = \frac{\dot{Q}_{comb}}{\dot{m}c_p} + T_2$$

All of these variables are known except $\dot{m}$. Will solve for $\dot{m}$ in part b and come back to finish finding T_3.

[1] $$T_3 = \frac{1500\ kJ/s}{(2.33\ldots\ kg/s)(1.07\ \frac{kJ}{kgK})} + (127+273)K \Rightarrow \boxed{T_3 = 1000K}$$

↖ $\dot{m}$ from part b

b) $\dot{m}$ is constant through each component. This question illustrates selecting the correct control volume. Consider the following control volume because information is known at its inlet and exit.



Assume $\Delta PE \approx \Delta KE \approx 0$

Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = \cancel{\frac{d\dot{E}_{cv}}{dt}}^{0} \Rightarrow \dot{E}_{in} = \dot{E}_{out}$

$$\dot{m}h_1 + \dot{W}_c + \dot{Q}_{comb} = \dot{W}_T + \dot{m}h_4$$

$$\dot{m}h_4 - \dot{m}h_1 = \cancel{\dot{W}_c - \dot{W}_T}^{0} + \dot{Q}_{comb}$$

$$\dot{m}(h_4 - h_1) = \dot{m}c_p(T_4 - T_1) = \dot{Q}_{comb}$$

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(b) cont.

[1.5] $\dot{m} = \dfrac{\dot{Q}_{comb}}{C_p(T_4 - T_1)} = \dfrac{1500\ \cancel{kJ}/s}{1.07\ \cancel{kJ}/(kg\cancel{K})((625+273)-(25+273))\cancel{K}} = 2.33645\ldots\ kg/s$

[1] $\boxed{\dot{m} = 2.34\ kg/s}$

(c) Energy Balance on Turbine: $\dot{E}_{in} - \dot{E}_{out} = \cancel{\dfrac{dE_{cv}}{dt}}^{0} \Rightarrow \dot{E}_{in} = \dot{E}_{OUT}$

$\dot{m}h_3 = \dot{W}_T + \dot{m}h_4$

[1.5] $\dot{W}_T = \dot{m}(h_3 - h_4) = \dot{m}C_p(T_3 - T_4) = \left(2.33\ldots\ \frac{kg}{s}\right)\left(1.07\ \frac{kJ}{kg\cancel{K}}\right)(1000 - (625+273))\cancel{K}$

[1] $\boxed{\dot{W}_T = 255\ \frac{kJ}{s} = 255\ kW}$

(d) At beginning of question, proved $\dot{m}$ constant throughout.

$\dot{m}_5 = \dot{m} = \rho_5 A_5 V_5 = \dfrac{A_5 V_5}{v_5}$

Because air is assumed an ideal gas, $P_5 v_5 = RT_5$

$v_5 = \dfrac{RT_5}{P_5}$

[1.5] $V_5 = \dfrac{\dot{m}v_5}{A_5} = \dfrac{\dot{m}RT_5}{P_5 A_5}$

$= \dfrac{\left(2.33\ldots\ \cancel{kg}/s\right)\left(0.287\ \frac{\cancel{kJ}}{\cancel{kg}\cancel{K}}\right)(25+273)\cancel{K}}{101.3\ \cancel{kPa}\ (30\cancel{cm^2})\left(\frac{1\cancel{m}}{100\cancel{cm}}\right)^2\left(\frac{1\cancel{kN}}{1\cancel{kPa}\,m^2}\right)} \cdot \left(\frac{1\cancel{kN}\ m}{1\cancel{kJ}}\right)$

[1] $V_5 = 657.542\ldots\ m/s$

$\boxed{V_5 \approx 658\ m/s}$

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Question #3. (concepts – answer briefly in the space provided)

[2.5] a) Is a thermodynamic system always defined by two thermodynamic properties always? No. A simple, compressible system is completely defined by two independent, intensive properties. (State Postulate)

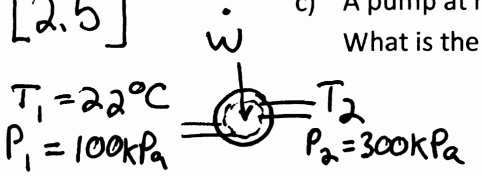
[2.5] b) Using the ideal gas relations to calculate from P and T the volume, V, near the top of the two phase dome of a real substance will result in an overestimate or underestimate of the volume? How did you reach this conclusion?

$$\frac{P\forall}{mRT} = Z \qquad \begin{array}{l} Z = 1 \text{ for an ideal gas} \\ Z < 1 \text{ for a real substance} \end{array}$$

$\forall = mRTz/P$ for a real substance.

Since $z<1$, if one assumes $z=1$ when it does not, will result in an $\boxed{\text{overestimate}}$ of the volume.

[2.5] c) A pump at room temperature, 22 C, pushes 5 l of water from 100 kPa to 300 kPa. What is the minimal power that needs to be supplied to the pump?



Assumptions for Pump:
1. Steady flow
2. $\Delta T = 0$
3. Water is incompressible ($dv = 0$)
4. $\dot{Q} = 0$
5. $\Delta PE \approx \Delta KE = 0$

Mass balance: $\dot{m}_{in} - \dot{m}_{out} = \cancel{\frac{dm_{cv}}{dt}}^{0} \Rightarrow \dot{m}_{in} = \dot{m}_{out} = \dot{m}$

Energy balance: $\dot{E}_{in} - \dot{E}_{out} = \cancel{\frac{dE_{cv}}{dt}}^{0} \Rightarrow \dot{E}_{in} = \dot{E}_{out}$

$\dot{W} + \dot{m}h_1 = \dot{m}h_2$

(see last page for conclusion)

[2.5] d) 10 kJ of heat are delivered from an environment at 273 K to a system at 22 C. Does this violate the first law of thermo?

$\boxed{\text{NO}}$ The first law is merely an energy balance equation. This violates the 2nd Law of Thermodynamics.

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(c) (continued.)

$$\dot{W} = \dot{m}(h_2 - h_1) = \dot{m}\Delta h$$

Assumption 3

On a differential level, $dh = du + v\,dP + \cancel{P\,dv}$

0 Assumption 2

Integrating: $\Delta h = \underbrace{\cancel{C_{avg}\Delta T}}_{\text{"u"}} + v\Delta P$

Thus $\dot{W} = \dot{m}\Delta h = \dot{m} v \Delta P = \dot{m} v (P_2 - P_1)$

$v = \dfrac{\forall}{m}$ Because steady flow, $\dfrac{\dot{m}}{m} = \dfrac{1}{t}$

$$\dot{W} = \dot{m}\frac{\forall}{m}(P_2 - P_1) = \frac{\forall}{t}(P_2 - P_1)$$

$$W = \forall(P_2 - P_1) = (5\cancel{L})\left(\frac{1\cancel{m^3}}{1000\cancel{L}}\right)(300-100)\cancel{kPa}\left(\frac{1\cancel{kN}}{1\cancel{kPa}\cdot\cancel{m^2}}\right)\left(\frac{1kJ}{1\cancel{kN\,m}}\right)$$

$\boxed{W = 1kJ}$ ← Energy

$\boxed{\dot{W} = \dfrac{W}{t}}$ ← Power