

ME 132, Spring 2003, Quiz # 2

# 1	# 2	# 3	# 4	# 5	TOTAL
15	12	12	6	15	60

1. Three systems are described below by their ODE (input u , output y). All are started with zero initial conditions at $t = 0^-$, namely $y(0^-) = \dot{y}(0^-) = \ddot{y}(0^-) = 0$. A unit-step input is applied at $t = 0$. In each case, determine:

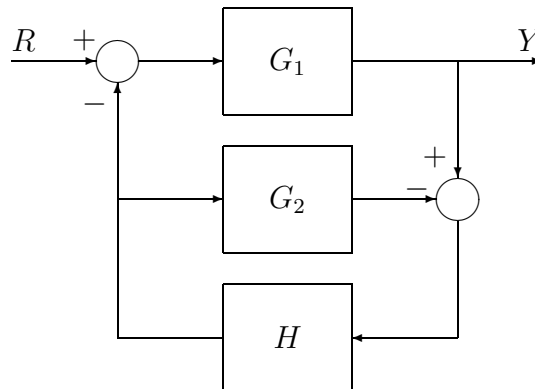
- the “new” initial conditions at $t = 0^+$, namely $y(0^+)$, $\dot{y}(0^+)$ and $\ddot{y}(0^+)$;
- the final value of y , ie., $\lim_{t \rightarrow \infty} y(t)$.

(a) $y^{[3]}(t) + 2y^{[2]}(t) + y^{[1]}(t) + y(t) = 6u^{[2]}(t) - 3u^{[1]}(t) + 2u(t)$

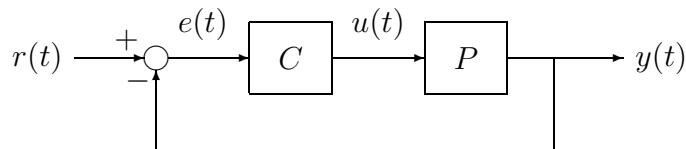
(b) $y^{[3]}(t) + 2y^{[2]}(t) + y^{[1]}(t) + 3y(t) = -3u^{[1]}(t) + 2u(t)$

(c) $y^{[3]}(t) + 2y^{[2]}(t) + 4y^{[1]}(t) + 5y(t) = 2u(t)$

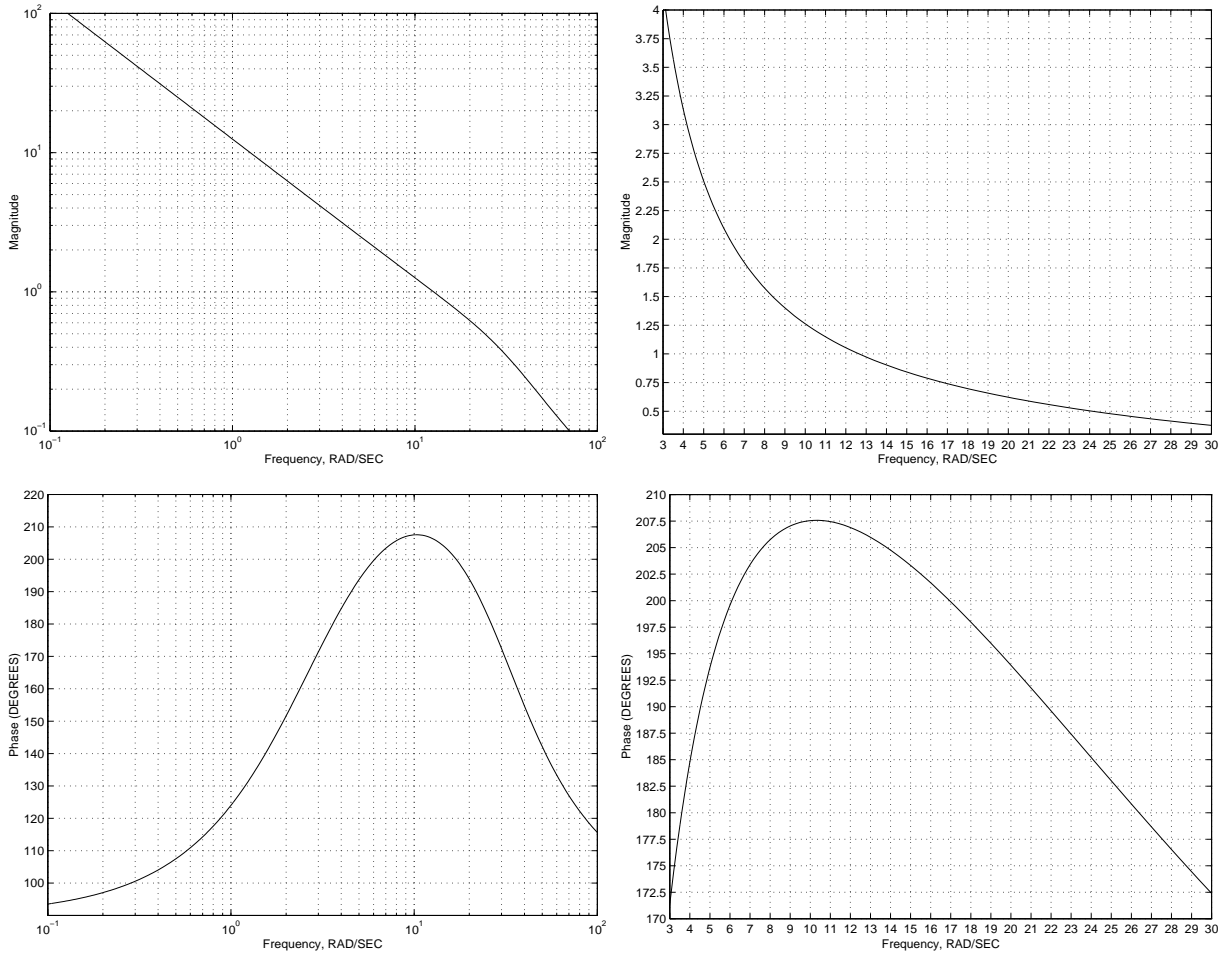
2. Assume G_1, G_2 and H are transfer functions of linear systems. Compute the transfer function from R to Y in the figure below.



3. A closed-loop feedback system consisting of plant P and controller C is shown below.



In this problem, it is known that the nominal closed-loop system is stable. The plots below are the magnitude and phase of the product $\hat{P}(j\omega)\hat{C}(j\omega)$, given both in linear and log scales, depending on which is easier for you to read. Use these graphs to compute the **time-delay margin** and the **gain margin**. Clearly indicate the gain-crossover and phase-crossover frequencies which you determine in these calculations.



4. A first order system has a transfer function

$$G(s) = \frac{\gamma}{\tau s + 1}$$

- (a) What is the differential equation relating the input and output?
 - (b) Under what conditions is the system stable?
 - (c) If the system is stable, what is the time-constant of the system?
5. Recall that if systems are connected in parallel (same input, and outputs add together) then the transfer function of the parallel connection is the sum of the transfer functions.

Consider the three different, complicated transfer functions

$$G_1(s) = \frac{0.05s^4 + 0.394s^3 + 7.868s^2 + 14.43s + 64}{0.04s^5 + 1.184s^4 + 7.379s^3 + 73.19s^2 + 95.36s + 64}$$

$$G_2(s) = \frac{0.05s^4 + 2.536s^3 + 64.36s^2 + 87.87s + 64}{0.04s^5 + 1.184s^4 + 7.379s^3 + 73.19s^2 + 95.36s + 64}$$

$$G_3(s) = \frac{0.9s^4 + 4.27s^3 + 65.97s^2 + 88.42s + 64}{0.04s^5 + 1.184s^4 + 7.379s^3 + 73.19s^2 + 95.36s + 64}$$

The step responses of all three systems are shown on the next page. The frequency responses (Magnitude and Phase) are also shown.

Although it may not be physically motivated, mathematically, each G_i can be decomposed additively as as (you do not need to verify this)

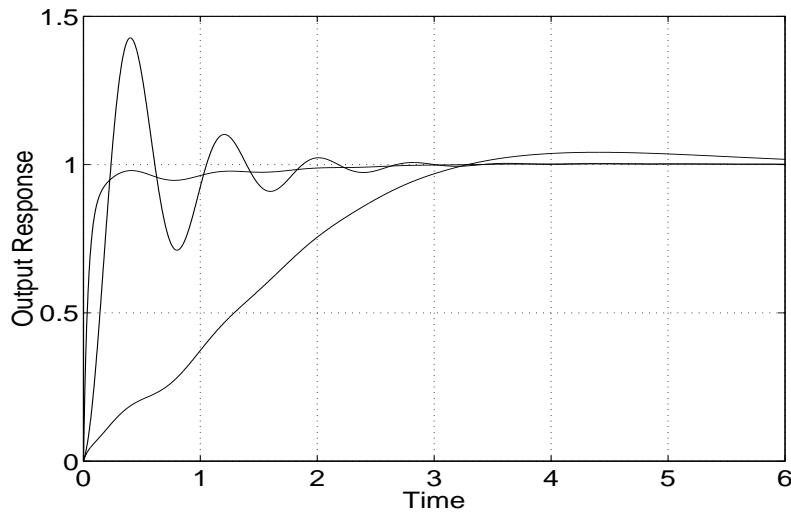
$$G_1(s) = 0.9 \frac{1}{s^2 + 1.4s + 1} + 0.05 \frac{64}{s^2 + 3.2s + 64} + 0.05 \frac{1}{0.04s + 1}$$

$$G_2(s) = 0.05 \frac{1}{s^2 + 1.4s + 1} + 0.9 \frac{64}{s^2 + 3.2s + 64} + 0.05 \frac{1}{0.04s + 1}$$

$$G_3(s) = 0.05 \frac{1}{s^2 + 1.4s + 1} + 0.05 \frac{64}{s^2 + 3.2s + 64} + 0.9 \frac{1}{0.04s + 1}$$

Based on this information, match up each G_i to its step response, frequency response magnitude, and frequency response phase.

Step Response of G_1, G_2, G_3



Frequency Response of G_1, G_2, G_3

