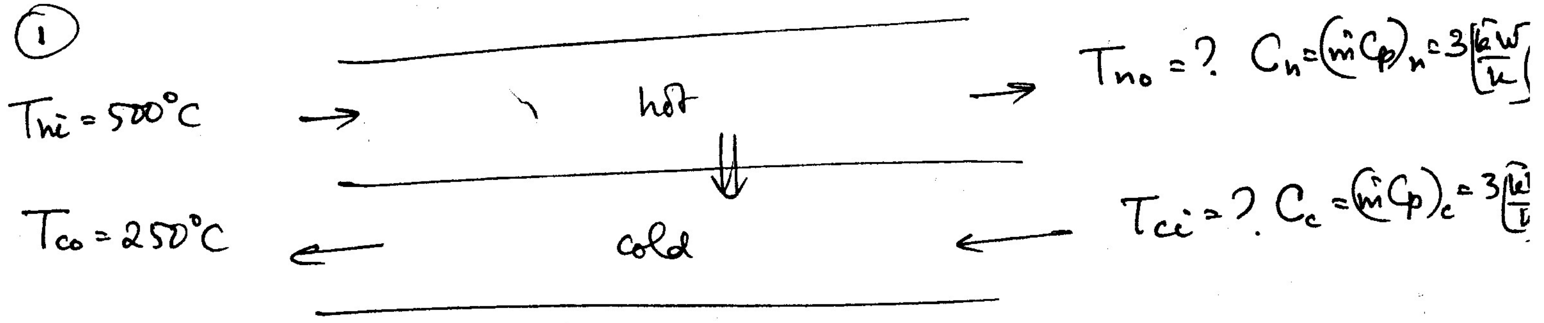
ME109 Midterm 2 Solutions

1



$$U = 3000 \ \frac{W}{m^2\text{-}K} \qquad A = 1\,m^2$$

$$UA = 3 \left[\frac{kW}{m^2\text{-}K}\right]$$

Use NTU Method

$$C_{min} = C_h = C_c = 3 \left[\frac{kW}{K}\right]$$

$$\boxed{1} \quad NTU = \frac{UA}{C_{min}}$$

$$NTU = \frac{3}{3} = 1$$

$$C_r = \frac{C_{min}}{C_{max}} = 1 \quad \boxed{1}$$

For $C_r = 1$ and counterflow HXer

$$\text{Effectiveness} \quad \epsilon = \frac{NTU}{1+NTU} = \frac{1}{1+1} = 0.5 \quad \boxed{1}$$

$$q_{max} = C_{min}\left(T_{hi} - T_{ci}\right) \quad \boxed{1}$$

$$q = \epsilon\, q_{max} = \epsilon\, C_{min}\left(T_{hi} - T_{ci}\right) \quad \boxed{1}$$

$$\text{Also} \quad q = C_c\left[T_{co} - T_{ci}\right] \quad \boxed{1}$$

Hence

$$\boxed{2} \quad C_c\left[T_{co} - T_{ci}\right] = \cancel{0.5}\ \epsilon\, C_{min}\left(T_{hi} - T_{ci}\right)$$

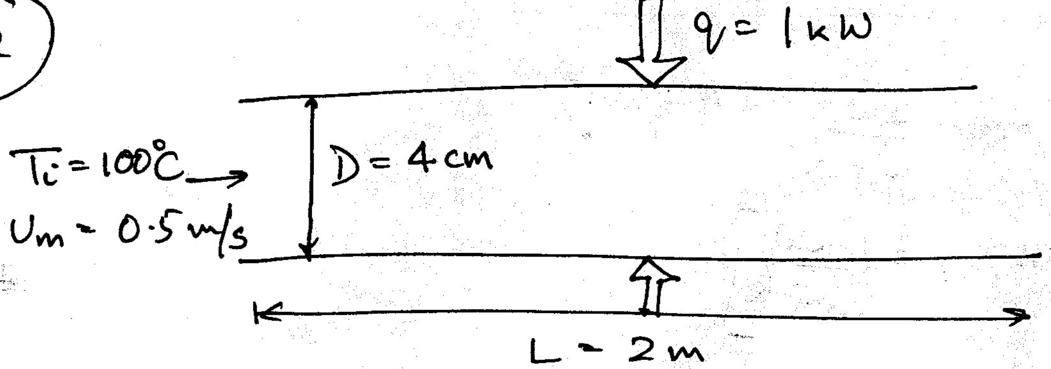
$$\cancel{3}(250 - T_{ci}) = 0.5 \times \cancel{3}(500 - T_{ci})$$

$$250 - T_{ci} = 250 - 0.5\,T_{ci} \quad \| \quad \boxed{T_{ci} = 0°C}$$

$$q = C_c\left(T_{co} - T_{ci}\right) = C_h\left(T_{hi} - T_{ho}\right)$$
$$= \cancel{\beta}\left(250 - 0\right) = \cancel{\beta}\left(500 - T_{ho}\right)$$ $\boxed{2}$

$$\boxed{T_{ho} = 250^\circ C}$$

2



(i) Mass flow rate $\dot{m} = \rho U_m A$ $\rho = 1\ kg/m^3$

$$\dot{m} = 1 \times 0.5 \times \pi(0.02)^2 = 6.3 \times 10^{-4}\ [kg/s]$$ $\boxed{1}$

(ii) Reynolds Number $Re_D = \frac{U_m D}{\nu}$ $\boxed{1}$ $\nu = 2 \times 10^{-5}\ [m^2/s]$

$$Re_D = \frac{0.5 \times 0.04}{2 \times 10^{-5}} = 1000$$ ← laminar flow

(iii) Energy Blance

$$\dot{m} C_p \frac{dT_m}{dx} = q'' P$$ $\boxed{1}$

$$\frac{dT_m}{dx} = \frac{q'' P}{\dot{m} C_p}$$

$\boxed{1}$ $$q'' = \frac{q}{\pi D L}$$ $P = \pi D$

$$\frac{dT}{dx} = \frac{q}{\pi D L} \times \frac{\pi D}{\dot{m} C_p} = \frac{q}{L \dot{m} C_p}$$

$$\cancel{T_\infty}\ T_m(x) = T_i + \frac{q\,x}{L\,\dot{m}\,C_p}$$

$$T_m(L) = T_i + \frac{q\,\cancel{L}}{\dot{m}\,C_p\,\cancel{L}} = T_i + \frac{q}{\dot{m}\,C_p}$$ $\boxed{1}$

Outlet temp $\rightarrow$ $$T_m(L) = 100\,(°C) + \frac{1\cancel{000}\ [W]}{6.3\times10^{-4}\ [kg/s] \times \cancel{1000}\ [J/kg\text{-}K]}$$

$$= 100°C + 1587.3\ °C$$

$$T_m(L) = 1687.3\ °C$$ $\boxed{1}$

(iv) $$q = hA\,(T_s - T_m)$$
$$q'' = h\,(T_s - T_m)$$ $\boxed{1}$

$$T_s = T_m + \frac{q''}{h}$$

$$Nu_D = \frac{hD}{k} = 4.36$$

$$h = \frac{4.36\times k}{D} = \frac{4.36\times 0.03}{0.04}$$

$$h = 3.27\ \left[\frac{W}{m^2\text{-}K}\right]$$ $\boxed{1}$

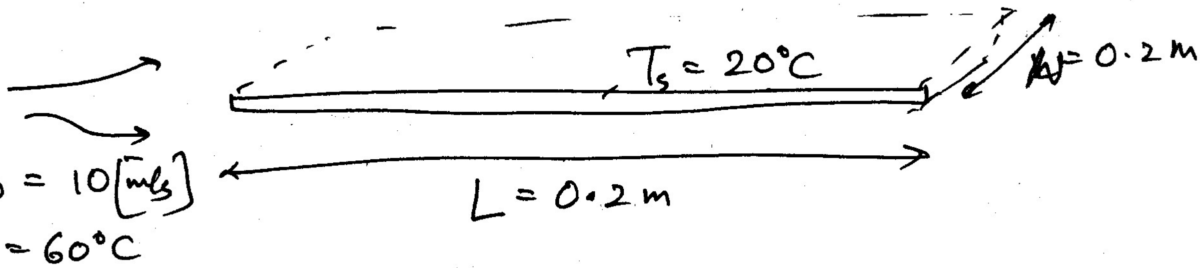
$$q'' = \frac{q}{\pi D L} = \frac{1000}{\pi\,(0.04)\times 2}$$

$$q'' = 3978.9\ [W/m^2]$$ $\boxed{1}$

$$T_s(L) = 1687.3 + \frac{3978.9}{3.27}$$

$$T_s(L) = 2904.09\ °C$$ $\boxed{1}$

3



$\boxed{1}$

$$Re_L = \frac{U_\infty L}{\nu} = \frac{10 \times 0.2}{2 \times 10^{-5}} = 10^5$$

Flow is always laminar

$$Pr = \frac{\nu}{\alpha} = \frac{\nu \rho C_p}{k}$$

$$Pr = \frac{2 \times 10^{-5} \times 1000}{0.03}$$

$$Pr = 0.67$$

$\boxed{1}$

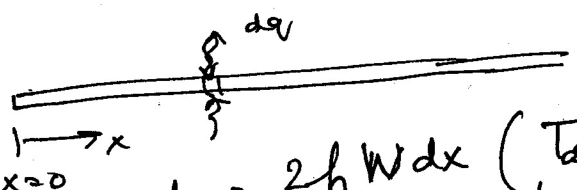
So $\boxed{2}$

$$Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$

$$\frac{h x}{k} = 0.332 \sqrt{\frac{U_\infty x}{\nu}}\; Pr^{1/3}$$

$$h = 0.332\, k \sqrt{\frac{U_\infty}{x \nu}}\; Pr^{1/3} = \frac{\phi}{\sqrt{x}}$$

$$\phi = 0.332\, k\, Pr^{1/3} \sqrt{\frac{U_\infty}{\nu}}$$



$$dq = 2 h W dx \,(T_\infty - T_s)$$ $\boxed{2}$

$\boxed{2}$

$$q = 2W(T_\infty - T_s) \int_0^L h\, dx$$

$$= 2W(T_\infty - T_s)\, \phi \int_0^L \frac{dx}{\sqrt{x}} = 4W(T_\infty - T_s)\, \phi \sqrt{L}$$

$$q = 4W(T_\infty - T_s)\; 0.332\, k\, Pr^{1/3} \sqrt{\frac{U_\infty L}{\nu}}$$

$\boxed{2}$

$$q = 4 \times 0.2\,(60 - 20)\; 0.332 \times 0.03\, (0.67)^{1/3} \sqrt{10^5}$$

$$q = 88.19\ [W]$$