MATH 142: EXAM 2

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1 (12 points). Consider the following four subspaces of $\mathbf{R}^2$:

$$A = \{(x,y) \in \mathbf{R}^2\,;\, x^2 + y^2 = 1\}$$
$$B = \{(x,y) \in \mathbf{R}^2\,;\, x^2 + y^2 \leq 1\}$$
$$C = \{(x,0) \in \mathbf{R}^2\,;\, -1 \leq x \leq 1\} \cup \{(0,y) \in \mathbf{R}^2\,;\, -1 \leq y \leq 1\}$$
$$D = \{(x,0) \in \mathbf{R}^2\,;\, 0 < x < 1\} \cup \{(0,y) \in \mathbf{R}^2\,;\, 0 < y < 1\}$$

The chart below contains a list of topological properties. For each property P and each space $X = A, B, C, D$ write **Yes** or **No** to indicate whether or not the space X has the property P. You do not need to explain your answers, although if you feel there is any ambiguity you should feel free to explain it.

	A	B	C	D
connected				
compact				
locally Euclidean (without boundary)				
simply connected				

2 (10 points). Let X be a metric space with metric

$$d : X \times X \to \mathbf{R}.$$

Recall that the *diamter* of a subset A of X is defined by

$$\operatorname{diam}(A) = \sup_{a,a' \in A} d(a,a').$$

Prove that if A is compact, then there exist $a, a' \in A$ such that

$$\operatorname{diam}(A) = d(a,a').$$

3 (14 points). Let X be a Hausdorff topological space and let A be a compact subset of X. Prove that A is closed in X.

4 (7 points). Prove that none of the three spaces

$$S^2, \qquad \mathbf{P}^2, \qquad S^1 \times S^1,$$

are homeomorphic to one another.

5 (7 points). Recall that a topological space X is said to be *locally compact* if every $x \in X$ has a neighborhood which is contained in a compact subset of X. We then define the *one-point compactification* X' of X to be the topological space which as a set is X together with a single point ∞; a subset $U \subseteq X'$ is open if either $\infty \notin U$ and U is open in X, or if $\infty \in U$ and there is a compact subset K of X such that $U = (X - K) \cup \{\infty\}$.

Prove that if X is connected and locally compact, then X is not homeomorphic to its one-point compactification X'.