Math 115 | Professor K. A. Ribet

Last Midterm Exam | November 3, 2000

This is a closed-book exam: no notes, books or calculators are allowed. Explain your answers in complete English sentences; no credit will be given for a "correct answer" that is not explained fully.

1 *(3 points).* Calculate the number of primitive roots mod $35035 = 5 \cdot 7^2 \cdot 11 \cdot 13$.

2 *(6 points).* What is the remainder when one divides the prime number 1234567891 by 11? What is the remainder when $11^{1234567890}$ is divided by 1234567891?

3 *(5 points).* Find a mod 29 inverse to the 2×2 matrix $\begin{pmatrix} 1 & 2 \\ 3 & 9 \end{pmatrix}$ mod 29.

4 *(9 points).* If p is a prime, show that all prime divisors of $2^p - 1$ are congruent to 1 mod p. (For example, $2^{11} - 1 = 23 \cdot 89$ is divisible by the primes 23 and 89 and by no others.)

5 *(7 points).* Let μ be the Möbius function, and let τ be the function whose value on $n \geq 1$ is the number of divisors of n. Explain why the function $F(n) := \sum_{d|n} \mu(d)\tau(d)$ satisfies the relation $F(n_1 n_2) = F(n_1)F(n_2)$ when $\gcd(n_1, n_2) = 1$. Calculate $F(p^e)$ when p is a prime and e is a positive integer.