ME 107A Midterm Solution

Spring 2008

1. With the help of a sketch and/or an equation when possible, define the following terms:

   a. Union and intersection of events (2 points)

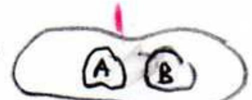
   Union : $P[A \cup B] = P[A] + P[B] - P[A \cap B]$ for overlaping event

   Intersection : $P[A \cap B]$ = hatched area

   b. Mutually exclusive events (2 points)

   $P[A \cap B] = 0$

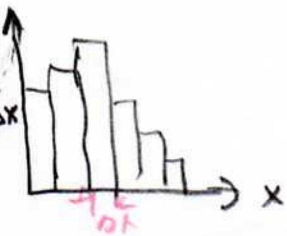
   mutually exclusive events

   c. Histogram (2 points)

   d. Frequency density distribution (2 points)

   $f_i^* = \frac{n_i}{N \cdot \Delta x}$

   e. Probability density function (2 points)

   Shows the probability of occurrence of a r.v. in a given interval

   f. Expected value (2 points)

   $$\mu = E[x] = \text{Expected value} = \text{mean value} = \int_{-\infty}^{+\infty} x f(x)\, dx$$

   g. Mean square (2 points)

   $$E[x^2] = \int_{-\infty}^{+\infty} x^2 f(x)\, dx \quad \text{or} \quad \overline{x}^2 = \sum_{i=1}^{n} \frac{x_i^2}{n}$$

   h. Standard deviation (2 points)

   $$\sigma = \left[\int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\, dx \right]^{1/2} \quad \text{or} \quad S_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}$$

   i. Skewness (2 points)

   $$\text{skewness} = \int_{-\infty}^{+\infty} (x-\mu)^3 f(x)\, dx \quad \text{or Non-dim. Skewness} = \frac{\text{Skewness}}{\sigma^3}$$

   j. Kurtosis (2 points)

   $$\text{Kurtosis} = \int_{-\infty}^{+\infty} (x-\mu)^4 f(x)\, dx \quad \text{or non-dim. divide by } \sigma^4$$

2. **A** – A sample of twelve measurements of temperature yielded a mean value 0f 20°C and a standard deviation of 0.3°C. Determine the 95% confidence interval for the estimated mean.

B – Determine the standard error and the precision uncertainty in the mean for the problem above.

1. A $\quad \nu = n-1 = 11 \qquad c = 0.95 \qquad \alpha = 1 - c = 0.05$

$$t_{\alpha/2,\,11} = 2.201$$

$$\bar{x} - 2.201\frac{S_x}{\sqrt{12}} < \mu < \bar{x} + 2.201\frac{S_x}{\sqrt{12}} \quad (95\%)$$

$\bar{x} = 20°C$ and $S_x = 0.3°C$

$$\boxed{19.81 < \mu < 20.19\ °C} \quad (95\%)$$

or $\mu = 20 \pm 0.19\ °C$

1. B Standard error $= \dfrac{S_x}{\sqrt{n}} = \dfrac{0.3}{\sqrt{12}}\ °C$

Precision uncertainty $= t_{\alpha/2,\,11}\dfrac{S_x}{\sqrt{n}} = 2.201\left(\dfrac{0.3}{\sqrt{12}}\right)°C \quad (95\%)$

3. A beam of length 3 ft. and a cross section area of 2 in^2 is subjected to axial tensile load. The beam is made of steel of yield strength = 40,000. lbf/in^2. If the standard deviation of the cross section area is 0.1 in^2 and of the yield strength is 2,000 lbf/in^2, determine the mean and standard deviation of the calculated yield load.

P_y = yield load $= A f_y \qquad f_y$ = yield strength

Mean value: $\bar{P}_y = \bar{A}\bar{f}_y = 2(40{,}000) = \underline{80{,}000}$ lbf

$$\sigma_P^2 \approx \left(\frac{\partial P_y}{\partial A}\right)^2_{\bar{P}_y}\sigma_A^2 + \left(\frac{\partial P_y}{\partial f_y}\right)^2_{\bar{P}_y}\sigma_f^2$$

$$= \bar{f}_y^2\,\sigma_A^2 + \bar{A}^2\,\sigma_f^2$$

$$= (40{,}000)^2(0.1)^2 + (2)^2(2{,}000)^2$$

$$= 32\,(10)^6\ (\text{lbf})^2$$

$$\sigma_P = 5660 \text{ lbf}$$