Math 115 — Professor K. A. Ribet

First Midterm Exam — February 25, 1998

Instructions: Answer question #2 and three other questions.

1 *(6 points)*. Find all solutions to the congruence $x^2 \equiv p \bmod p^2$ when p is a prime number.

2 *(9 points)*. Using the equation $7 \cdot 529 - 3 \cdot 1234 = 1$, find an integer x which satisfies the two congruences $x \equiv \begin{cases} 123 & \bmod 529 \\ 321 & \bmod 1234 \end{cases}$ and an integer y such that $7y \equiv 1 \bmod 1234$. (No need to simplify.)

3 *(7 points)*. Suppose that p is a prime number. Which of the $p+2$ numbers $\binom{p+1}{k}$ $(0 \le k \le p+1)$ are divisible by p? [Example: The seven binomial coefficients $\binom{6}{k}$ are 1, 6, 15, 20, 15, 6, 1; the middle three are divisible by 5.]

4 *(7 points)*. Let p be a prime and let n be a non-negative integer. Suppose that a is an integer prime to p. Show that $b := a^{p^n}$ satisfies $b \equiv a \bmod p$ and $b^{p-1} \equiv 1 \bmod p^{n+1}$.

5 *(6 points)*. Show that $n^4 + n^2 + 1$ is composite for all $n \ge 2$.