Math 115 Professor K. A. Ribet

First Midterm Exam September 29, 2000

This is a closed-book exam: no notes, books or calculators are allowed. Explain your answers in complete English sentences. No credit will be given for a "correct answer" that is not explained fully. In general, there is no need to simplify numerical answers.

1 *(5 points)*. Let a and b be positive integers for which a^4 divides b^3. Prove that a divides b.

2 *(10 points)*. Let $f(x) = x^2 - x - 1$. Here are some values of f:

i	0	1	2	3	4	5	6	7	8	9	10	$\cdots$
$f(i)$	-1	-1	1	5	11	19	29	41	55	71	89	$\cdots$.

Find integers a and b so that $f(a)$ and $f(b)$ are both divisible by 11^2 but so that $a - b$ is not divisible by 11^2. Find the number of solutions mod $5 \cdot 11^2$ to the congruence $f(x) \equiv 0 \bmod 5 \cdot 11^2$.

3 *(3 points)*. Let $m = 173 \cdot 193$. Find positive integers a and b with $\sqrt{m} < b < \dfrac{m+1}{2}$ for which $m = b^2 - a^2$.

4 *(5 points)*. Use the identity

$$1 = 89 \cdot 24 - 61 \cdot 35 \qquad (*)$$

to solve the simultaneous congruences

$$x \equiv \begin{cases} 3 & \bmod 89 \\ 12 & \bmod 61. \end{cases}$$

5 *(4 points)*. Using (*), find integers a and b with $1 = 24a + 35b$ and $|a|$ as small as possible.

6 *(3 points)*. Using (*) yet again, solve the congruence $35x \equiv 2 \bmod 89$.