Math 115 Professor K. A. Ribet

First Midterm Exam, February 22, 1994

1. Is 270 a square modulo the prime number 691?

2. The decimal expansion of $1/7$ is $0.\overline{142857} = .142857142857\ldots$. Find all prime numbers p for which the decimal expansion of $1/p$ has period six. [It may help to know that $99 = 3^2 \cdot 11$, $999 = 3^3 \cdot 37$, $9999 = 3^2 \cdot 11 \cdot 101$, $99999 = 3^2 \cdot 41 \cdot 271$, and $999999 = 3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$.]

3. Using the Euclidean algorithm, find integers n and m such that $13n + 47m = 1$.

4. Let n be a positive integer. Calculate the limit $\lim_{k\to\infty} \dfrac{n^{k+1}}{\sigma(n^k)}$, where σ denotes, as usual, the function whose value at m is the sum of the divisors of m.

5. Show that there are an infinite number of primes which are congruent to 7 mod 8. [If $P_1, \ldots, P_n$ are such primes, consider $(P_1 \cdots P_n)^2 - 2$.]

6. Let n be a positive integer. Show that $2^n \equiv 1 \bmod n$ if and only if $n = 1$. [For $n > 1$, consider the situation modulo the smallest prime number dividing n.]