

Math 115  
Last Midterm Exam

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☞ The numbers 257 and 661 are prime.

**1** (5 points). Find the number of square roots of 9 modulo  $3 \cdot 11^2 \cdot 13^3$ .

**2** (5 points). Determine whether or not 116 is a square modulo 661.

**3** (5 points). Determine whether or not 116 is a cube modulo 661.

**4** (5 points). Calculate the number of primitive roots modulo  $257^2$ .

**5** (7 points). Express  $-\frac{15}{47}$  as a continued fraction.

**6** (8 points). Let  $p$  be a prime number dividing  $x^2 + 1$ , where  $x$  is an even integer. Show that  $p \equiv 1 \pmod{4}$  and that  $p$  is prime to  $x$ . Deduce that there are an infinite number of primes congruent to 1 mod 4.